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**Sustained Growth Driven
by Multiple, Co-Existing
GPTs**

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SUSTAINED GROWTH DRIVEN BY MULTIPLE, CO-EXISTING GPTs¹**4 September 2007**

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ABSTRACT

The model incorporates characteristics of general purpose technologies established empirically but not currently modeled: GPTs occur simultaneously in several technology “classes,” such as ICTs and materials; different “*versions*” of each class often compete with each other; GPTs of different classes complement each other; uncertainty is associated with GPT development and diffusion. The model’s three sectors produce consumption goods using applied knowledge, applied knowledge using GPTs, and pure knowledge that occasionally discovers a new GPT whose efficiency increases as it diffuses. The model allows for competition between, and complementarities among GPTs, replicates accepted growth facts and is useful for policy analysis.

TABLE OF CONTENTS

SUSTAINED GROWTH DRIVEN BY MULTIPLE, CO-EXISTING GPTS.....	1
I. THE SEQUENTIAL GPT MODEL.....	5
II. THE SIMULTANEOUS GPT MODEL OUTLINED.....	8
SOME TERMINOLOGY	9
RELATIONS AMONG GPTS	10
THE PURE KNOWLEDGE SECTOR.....	12
IV THE MODEL SIMULATED.....	17
V THE MODEL APPLIED.....	19
APPENDIX.....	23
MEETING THE STYLIZED FACTS OF ECONOMIC GROWTH.....	23
I. KALDOR'S FACTS	23
II. EASTERLY AND LEVINE	27
TEXT REFERENCES	30

Over the last several years the concept of general purpose technologies (GPTs)² has been applied to many issues in economic history, industrial organization and economic policy.³ These applications have been restricted by the nature of the first generation of GPT models found in Elhanan Helpman (1998), which cover only the evolution of a single GPT that dominates the macro performance of the whole economy. This has led, for example, to attempts to infer the existence of a single GPT by examining the aggregate behaviour of the economy as measured by its GDP.

In spite of the need for more empirically relevant models, there have been very few further developments of GPT theory using standard techniques.⁴ One reason for this is that because these models all use dynamically stationary equilibrium concepts in which agents necessarily foresee the behaviour of a new GPT over its whole lifetime and maximize over that horizon, the models soon become analytically intractable when more empirically relevant assumptions are introduced. We have circumvented this modeling problem in Lipsey Carlaw and Bekar (2005, hereafter LCB) and in Carlaw and Lipsey (2006) by building dynamically non-stationary equilibrium models of GPT-driven growth that are analytically manageable because we forgo the use of a stationary foresighted equilibrium. In our models, agents face a future that is uncertain and do the best they can with limited knowledge of current relations. This assumption allows us to introduce more complex and realistic assumptions concerning the structure of technology and the behaviour of agents.

The model presented in this paper, incorporates a number of characteristics of general purpose technologies that have been established by historical research. Only point (1) and a few of the sources of uncertainty listed in point (6) below have been modeled so far.

1. The efficiency with which any new GPT delivers its services increases greatly over time.
2. The use of any new GPT spreads slowly through the economy and its full diffusion with extension to many different uses typically takes decades.
3. GPTs occur in each of several “*classes*” of technology, such as materials, ICTs, power sources, transportation equipment, and organizational forms

² “A GPT is a single generic technology, recognizable as such over its whole lifetime, that initially has much scope for improvement and eventually comes to be widely used, to have multiple uses, and to have many spillover effects.” Lipsey Carlaw and Bekar (2005: 98)

³ For examples, in economic history see Nicholas Crafts (2003), David Greasely and Les Oxley (1997), Joel Mokyr (2006) and Vernon W. Ruttan (2005); in applied microeconomics see Andreas Hornstein, Per Krusell, and Giovanni L. Violante (2006) Lindmark (2005) and Sandro Sapio and Grid L. D. Thoma (2006); in assessments of policy see Chris Forman, Avi Goldfarb, and Shane Greenstein (2003), A. Edward Safarian and Wendy Dobson (2002), Adreaan van Zon and Tobias Kronenberg (2005) and Roger Wehrli and Gay Saxby (2006); in practical policy strategies that directly employ the concept, see New Zealand’s development of two entire policy frameworks in *The Growth and Innovation Framework* (GIF) and the *Digital Strategy*. (see, <http://www.digitalstrategy.govt.nz/>) as well as the Australian Research Council, (2006).

⁴ One example of these further developments is Adriaan van Zon, Emmanuelle Fortune and Tobias Kronenberg (2003)

(e.g., the factory system) and at least one version of each class is in use at any one time.

4. Over time, many different “*versions*” of each class are invented. These often compete with each other and, as a result, there can be several versions of any one class in simultaneous use. For example, in 1900 some textile factories were shifting to electricity as a power source, while most were steam powered, and not a few were still using water wheels—three versions of GPTs all within the class power delivery technologies.⁵
5. In contrast, GPTs of different classes often complement each other, as when electricity enabled electronic computers and lasers.
6. There are many sources of uncertainty (in Frank Knight’s meaning of the term) in invention and innovation with respect to any new technology including GPTs. In particular, the following things are uncertain: (i) how much potentially useful pure knowledge will be discovered by any given amount of research activity; (ii) the timing of the discovery of new technologies; (iii) just how productive a newly innovated GPT will be over its lifetime; (iv) how well the new GPT will interact with GPTs of other classes that are also in use; (v) how long a new GPT will continue to evolve in usefulness; (vi) when it will begin to be replaced by a new superior version of a GPT of the same class (vii) how long that displacement will take and (viii) if the displacement will be more or less complete (as were mechanical calculators) or if the older technology will remain entrenched in particular niches (as does steam that remains an important source of power for generating electricity).

We couch our subsequent discussions in terms of GPTs because we believe that they are important concepts for understanding the behaviour of Western technological achievements over the last 10,000 years. This, however, is only a verbal interpretation of our model whose key structure is that new knowledge is produced at two different levels: a pure research level that produces fundamental ideas and an applied research level that uses these ideas to produce applications useful to both the consumption goods industries and the pure research sector. The pure research sector could be producing potential GPTs, as in our verbal interpretations of the model, or it could be producing knowledge that is less universal in its application as long as it provides fodder for the applied R&D sector to produce useful applications. In another interpretation, it could be producing knowledge that allows GPTs developed abroad to be used to produce applications in the domestic economy, allowing the model to be applied wherever countries are adopting and adapting technologies developed elsewhere.⁶

I. THE SEQUENTIAL GPT MODEL

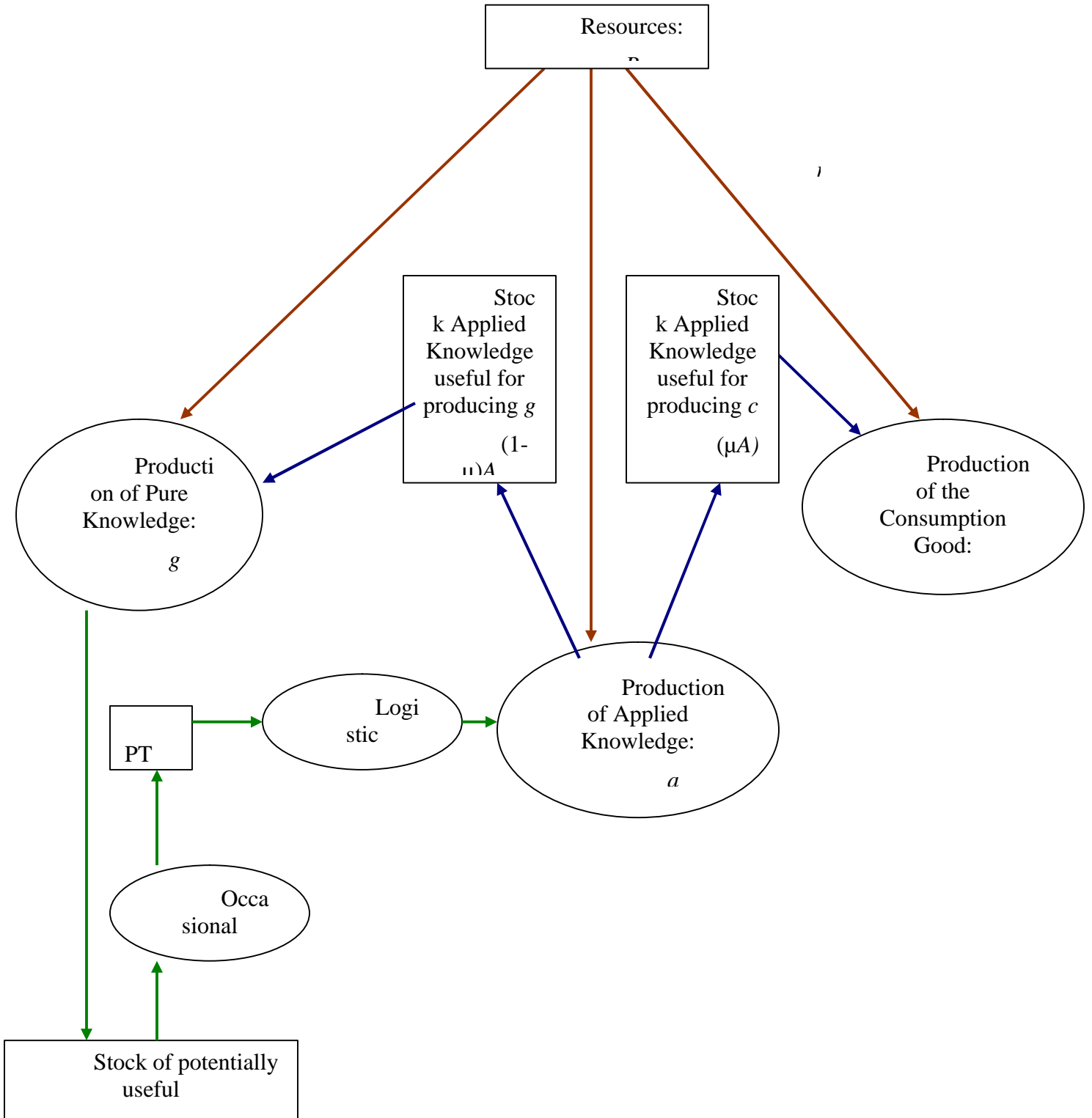
⁵ Some informal models that are expressed in verbal terms do deal with multiple GPTs.

⁶ Wesley M. Cohen and Daniel A. Leventhal (1990) call this knowledge producing activity the building of “absorptive capacity”.

This paper uses the basic structure common to our previous models in LCB, (Chapters 14 and 15) and Carlaw and Lipsey (2006). As shown in Figure 1, the model has three sectors. One sector produces pure research that occasionally discovers a new GPT; one sector produces applied research that develops applications for the GPT; one sector produces a consumption good using the results of applied research in its production

function. Each sector has its own distinct aggregate production function. Thus, the intra-sector technology is flat, while a technology structure is imposed through the inter-sector relations among these three different production functions.

FIGURE 1



The economy has a fixed aggregate stock of a composite resource, R , which can be thought of as ‘land’ and ‘labour.’ The three aggregate production functions display diminishing marginal returns to this resource (capital being held constant). The pure knowledge sector produces a flow of pure knowledge, g , which accumulates in a stock of potentially useful knowledge, Ω . Every once in a while a new GPT is invented. The existing stock of potentially useful pure knowledge is embodied in it and then, as in LCB (2005), its efficiency slowly evolves according to a logistic function to become increasingly useful in applied research.⁷ The applied R&D sector produces practical knowledge that is useful in both the consumption and the pure research sectors, the latter being a feedback that is well established in the technology literature.⁸ This knowledge is embodied in physical and human capital, which at this level of aggregation we do not need to treat separately. The stock of applied knowledge is *divided* between a proportion, m that is useful in the consumption industry and, $1-m$ that is useful in further pure research. The proportion that is useful to the consumption sector enters its aggregate production function as a productivity coefficient with constant returns to scale. Thus the consumption sector’s aggregate production function is of the form $C = mA(f_c)^a$ where m is the proportion of applied knowledge embodied in capital that is useful in the consumption sector, f_c is the amount of the composite resource that is allocated to the consumption sector and $a \in (0,1)$.

Adopting the assumption that applied knowledge is divided between what is useful in the consumption sector and in the pure knowledge sector ensures that we do not introduce increasing returns to accumulating applied knowledge, nor any externalities.⁹ Among other things, this allows us to produce a model of sustained endogenous growth without some of the characteristics that are often needed to sustain growth in other models of endogenous growth. It also creates a model suited to studying the relation between technological change and standard measures of changes in TFP, calculations of which are usually based on the assumption of constant returns to the accumulating factors.

The model produces endogenously determined sustained growth at a rate that varies over time.

II. THE SIMULTANEOUS GPT MODEL OUTLINED

In this section, we discuss our amendments to the above model which are designed to incorporate the first five numbered characteristics listed in the introduction while the various sources of uncertainty outlined in point 6 are introduced as we specify our model formally in the next section. To allow for different classes of GPTs (characteristic 3), we introduce multiple activities in each of our three sectors. The pure research sector has X laboratory complexes, which we call ‘labs’ for short, each producing a distinct class of pure knowledge and making use of all types of applied

⁷ In Carlaw and Lipsey (2006) which was written before LCB (2005) but published later, the full potential of a new GPT is achieved instantly.

⁸ See, for example, LCB, (Chapter 4) and Nathan Rosenberg (1982: Chapter 7).

⁹ If we had assumed that the whole stock of applied knowledge was useful in the consumption sector while some fraction of it spilled over as an externality to increase the efficiency of pure research, we would have introduced increasing returns to knowledge.

knowledge. Each lab occasionally invents a new version of a GPT in its particular class. The applied R&D sector has Y ‘facilities,’ each producing a distinct type of useful knowledge, and employing one version of each class of GPT. The consumption goods sector has I industries, each producing a different consumption good, and making use of the stocks of applied knowledge. The efficiency of each GPT, $(G_{n_x})_t$, evolves according to a logistic function that is given in equation (8) below, until its full potential has been realized (characteristic 1). This increasing productivity affects both the applied R&D facilities that are currently using the GPT and the GPT’s potential were it to be adapted by other facilities at some future time. In order to allow a new GPT to spread through the economy over time (characteristic 2), the productivity of each GPT is allowed to differ in each of the applied R&D facilities. This is done by multiplying each GPT’s productivity by a parameter θ that is specific both to that GPT and the R&D facility in which it is operating.

Each R&D facility is initially seeded with one version of each class of GPT. When a new GPT is invented—we call it a ‘challenger’—each applied R&D facility must decide whether to adopt it or to stay with the version of the GPT of that class that it is presently using—we call this the ‘incumbent.’ Because of the uncertainties listed in (6) above, the full evolution of the incumbent and challenging GPTs cannot be predicted in advance and then compared. So some more restricted choice criteria are needed. Of the many possibilities, we use a simple comparison of the challenger’s current level of productivity with the current productivity of the incumbent.¹⁰ If a new challenging GPT has a lower current productivity than the incumbent in all applied R&D facilities, it is sent back to the pure research sector for further development and it is reconsidered for adoption in every subsequent period. If it has a higher productivity in one or more facilities, it is adopted there. Each subsequent period, the facilities that have not adopted this new GPT compare its evolving productivity with the evolving productivity of their incumbent and switch when the former exceeds that latter. Over time, the use of a new version of any one class of GPT will spread through the economy, as one applied R&D facility after another adopts it and discards its incumbent. This activity models characteristic 4 and the activity that underlies LCB’s applications curve, (LBC: 436). Thus, there may be several versions of GPT of any one class in use at any one time. Each period, each facility that is not already using the latest version of any one class of GPT compares the productivity of its incumbent with the productivities of all existing newer versions of GPTs in that class. It may stay with the incumbent or adopt some newer version. It is possible, however, that another version of GPT of that class will be invented before the first challenger has been taken up by all applied R&D facilities.

Some Terminology

The variable x indicates a specific *class* of GPT. The index n_x identifies a GPT in the sequence of *versions*, $1..n$, of GPTs in class x that has been invented and adopted by at least one applied R&D facility. At any one time, the latest version adopted in that class is denoted by n_x and the previously adopted version by $(n - 1)_x$. We refer to the productivity

¹⁰ In LCB we study the effects of five different adoption criteria.

of the most recently invented GPT from lab x at time t as $(G_{n_x})_t$ and the previously invented GPT in that class as $(G_{(n-1)_x})_t$. The variable t_{n_x} refers to the invention date of the latest version of the class of GPT invented by lab x , while $t_{(n-1)_x}$ refers to the invention date of the previous version from lab x and $(t-1)_{n_x}$ refers to the period just prior to the invention of GPT of version n_x .

Relations Among GPTs

The productivity coefficient in each applied R&D facility is the geometric mean of the productivities of the GPTs that it uses (one version from each class of GPT), each pre-multiplied by an associated θ value (see equation (2) below). In our treatment, the θ 's vary randomly over a small range centered on unity.

The θ 's can be set out in a $Y \times X$ matrix where a row indicates the class of GPT and a column the research facility. We call this the 'operative θ matrix'. Initially all the θ 's in this matrix are set at unity. Thus the initial GPT in each class has the same level of productivity in each of the applied R&D facilities and the GPTs work together in each of the facilities according to their unmodified productivities. When a new version of a particular class of GPT is invented, it brings with it its own matrix of potential new θ 's, which we call its 'potential θ matrix'. When each R&D facility decides whether or not to adopt the challenger, it must form expectations over the potential θ 's required to make the calculation of what the output would be if the challenger were used. We denote these expected values by $\bar{\theta}$. Because the $\bar{\theta}$'s vary across any row, different facilities will evaluate the productivity of the challenger differently and so some may adopt it while others do not. When a particular applied R&D facility adopts the challenger, the challenger's potential θ 's replace the existing ones in that facility's column vector in the operative θ matrix. The resulting change in the θ 's associated with GPTs in classes other than the newly adopted GPT indicates whether the new GPT cooperates better or worse with the existing GPTs of other types than did the replaced incumbent (characteristic 5). As time goes by, the operative matrix of θ 's changes and at any one time each column in the matrix is derived from the potential θ 's of the latest GPT to be adopted by that R&D facility.

The full identification of each θ requires four indexes, $\theta_{y,z}^{n_x}$. The superscript n_x tells us that the θ is modifying a GPT of class x , version n . The subscript tells us that the GPT is being used by research facility y and replaced the previous v when a new version of class z GPT was adopted. Thus, z indicates the class of GPT that was last adopted by that facility and it is the source of all the θ 's in that column, while n_x indicates the version of class- x GPT to which the θ is being applied. Note that the x in the superscript and the y in the subscript do not change over time, the former indicating the class of GPT being referred to and the latter the R&D facility that is using it. In contrast, the n and z do change over time, the n indicating the version of the particular class- x GPT being used by the facility in question and the z indicating the class of the last GPT adopted by that facility (and hence the source of all the θ 's in that facility's column vector).

An example may help to illustrate how the θ 's work. Let there be three classes of GPT, developed by three pure research labs, $x = 1, 2, 3$, and three types of applied R&D

facility, $y = 1, 2, 3$. The nine individual n s can be displayed in a 3x3 matrix. Each column modifies the productivities of each of the GPTs used in one of the applied R&D facilities. Each row modifies the productivities of the particular version of a specific class- x GPT used in each of the applied R&D facilities.

As new GPTs are invented and adopted the operative matrix of ρ s evolves, the relevant column vector changing each time a new version of one type of GPT is adopted in a particular facility. Table 1 shows an illustrative operative matrix that might have evolved after several inventions and adoptions of new GPTs.

TABLE 1
ILLUSTRATIVE DISTRIBUTION OF n s FOR A MODEL
WITH THREE GPTs AND THREE APPLIED R&D FACILITIES

		$y =$		
		1	2	3
$x =$	1	$n_{1,1}^3$	$n_{2,1}^4$	$n_{3,3}^2$
	2	$n_{1,1}^2$	$n_{2,1}^2$	$n_{3,3}^1$
	3	$n_{1,1}^1$	$n_{2,1}^1$	$n_{3,3}^3$

The first column in the table tells us that the source of the ρ s in the first R&D facility was the third version of class 1 GPT, which the facility is using along with the second version of class 2 GPT and the first version of class 3. The second column tells us that the source of the ρ s in that column was the fourth version of class 1 GPT. (Evidently that version was rejected by facility 1, which is still using the third version). This GPT is being used in combination with the second version of class 2 GPT and the first version of the class 3 GPT. The third column tells us that the source of the ρ s for the third applied R&D facility was the second version of class 3 GPT, while the facility is using that GPT along with the second version of class 1 and the first version of class 2 GPTs.¹¹

The ρ s can be determined randomly, as we do in our present theoretical treatment, or they can be set so as to model any desired degree of competitiveness among different versions of GPTs of any one class, and different amounts of complementarity among GPTs of different classes operating together in any one R&D facility.

III THE MODEL SPECIFIED

The fixed supply of the composite resource, R , is allocated by private price-taking agents in the consumption and applied R&D sectors and by a government that taxes the applied R&D and consumption sectors to fund pure research at an exogenously determined level.¹² We make the following assumptions about agent behaviour. (1) Agents are price takers. (2) Agents are operating under conditions of Knightian uncertainty for all of the reasons set out in characteristic 6, Section I, which we model by

¹¹ As a check, if a second version of GPT class 2 is invented and adopted by R&D lab 1, the first column in the operative matrix alters to read: $n_{1,1}^3; n_{1,2}^2; n_{1,2}^1$.

¹² In Section VI below we change this allocation behavior to analyze R&D support policy.

assuming they do not know the probability distributions that are generating the disturbances on the outcomes. (3) Because agents cannot assign probabilities to alternative future consumption payoffs, they seek to maximize their profits on the basis of current prices.

The constraint imposed by the composite resource is:

$$(1) \quad R_t = \sum_{i=1}^I r_t^i + \sum_{y=1}^Y r_t^y + \sum_{x=1}^X r_t^x$$

The Applied R&D and the Consumption Sectors

The output of applied knowledge from each applied R&D facility, y , depends on the amount of the composite resource it uses and its productivity coefficient, which is the geometric mean of each $(G_{n_x})_t$ term multiplied by its corresponding v term, as shown in equation (2).

$$(2) \quad a_t^y = \left[\prod_{x=1}^X (\mathbf{n}_{y,z}^{n_x} (G_{n_x})_{t-1})^{b_x} \right]^{\frac{1}{X}} (r_t^y)^{b_{X+1}}$$

$$\mathbf{b}_x \in (0,1] \quad \forall x \in X, \quad \mathbf{b}_{X+1} \in (0,1)$$

The stock of applied knowledge generated from each facility accumulates according to:

$$(3) \quad A_t^y = a_t^y + (1 - \mathbf{e})A_{t-1}^y,$$

where $\mathbf{e} \in (0,1)$ is a depreciation parameter.

In the consumption sector, we make the simplifying assumptions (1) that there are the same number of applied R&D facilities and consumption industries, $Y = I$, and (2) that the knowledge produced in each of the facilities, y , is useful only in the one corresponding consumption industry, i .¹³ The production function for each of the I industries in the consumption sector is then expressed as follows:

$$(4) \quad c_t^i = (\mathbf{m}A_{t-1}^y)^{a_y} (r_t^i)^{a_{Y+1}}, \quad \mathbf{a}_y \in (0,1] \quad \forall y \in Y, \quad \mathbf{a}_{Y+1} \in (0,1) \quad \text{and } i = y$$

The Pure Knowledge Sector

As already observed, there are X labs each producing one class of pure knowledge that leads to the occasional invention of a new version, n_x , of that class of GPT. The productivity coefficient in each lab is the geometric mean of the various amounts of the Y

¹³ This assumption greatly simplifies the calculations and is adequate for present purposes. In some specific applications, we need to make each consumption industry use more than one type of applied knowledge. At the extreme, if each uses all types, their production functions become:

$$c_t^i = \left[\prod_{y=1}^Y (\mathbf{m}A_{t-1}^y)^{a_y} \right]^{\frac{1}{Y}} (r_t^i)^{a_{Y+1}}, \quad \mathbf{a}_y \in (0,1] \quad \forall y \in Y, \quad \mathbf{a}_{Y+1} \in (0,1).$$

different kinds of applied knowledge that are useful in further pure research (one for each applied R&D facility and each raised to a power \mathbf{s}_y).¹⁴ The output of pure knowledge in lab x , g_t^x , is a function of this productivity coefficient and the amount of the composite resource devoted to that lab.

$$(5) \quad g_t^x = \left[\prod_{y=1}^Y \left((1-\mathbf{m}) A_{t-1}^y \right)^{\mathbf{s}_y} \right]^{\frac{1}{Y}} \left(\mathbf{q}_t^x r_t^x \right)^{\mathbf{s}_{Y+1}},$$

$$\mathbf{s}_y \in (0,1], \quad \forall y \in Y \text{ and } \mathbf{s}_{Y+1} \in (0,1).$$

The term, \mathbf{q}_t^x , models the uncertainty surrounding the productivity of the composite resource devoted to pure research as indicated in uncertainty source 6 (i) listed in Section I.

The stocks of *potentially useful* knowledge produced by each of the X labs accumulate according to:

$$(6) \quad \Omega_t^x = g_t^x + (1-\mathbf{d})\Omega_{t-1}^x$$

where $\mathbf{d} \in (0,1)$ is a depreciation parameter.

New GPTs are invented infrequently in each of the X labs and their invention date is determined when the drawing of the random variable $\mathbf{I}_t^x \geq \mathbf{I}^{*x}$ (uncertainty source 6 (ii)). For simplicity, we let the critical value of lambda for each of the X labs be the same: $\mathbf{I}^{*x} = \mathbf{I}^* \quad \forall x \in X$. When at any time, t , $\mathbf{I}_t^x \geq \mathbf{I}^*$, indicating that a new version of class- x GPT is invented, the index t_{n_x} is reset to equal the current t , and n_x is augmented by one.

Because agents do not know how productive a new GPT will be over its lifetime (uncertainty source 6 (iii)), they must make their adoption decisions with incomplete information. In this paper, as discussed in Section III, we use the rule that the class- x challenger is adopted wherever its initial productivity is expected to exceed that of the class- x incumbent (determined according to equation (2), using the operative ν matrix). The expected productivity from using the challenger is determined from its new $(G_{n_x})_t$ term defined in equation (8) below, the productivities of the other classes of GPT used by that facility and the challenger's expected $\bar{\mathbf{n}}$'s, which alter those productivities. We assume that agents considering adopting the new GPT can correctly predict the potential ν associated with the version of the challenger, the minimum knowledge that they need to make some kind of evidenced-based adoption decision, but that they predict the other $\bar{\mathbf{n}}$'s in their facility's column vector will be unchanged. In other words, they assume that the challenging GPT will cooperate with the other GPTs used in the facility with the same level of efficiency as did the incumbent, a prediction that may be falsified since the potential ν 's brought in by the challenger may differ from those in the current

¹⁴ We continue with the assumption first made for our one-at-a-time GPT models, and discussed in Section II above, that applied knowledge is divided into a proportion, \mathbf{m} that is useful in the consumption industries and a proportion, $(1-\mathbf{m})$, that is useful in pure knowledge production.

operative matrix (uncertainty source 6(iv)). For example, in the case illustrated in Table 1, if facility 1 is considering a challenger of class 2, it expects that the ? in cell (1, 2) will become the potential $\bar{n}_{1,2}^{3_2}$ associated with the challenger but that the ?s in cells (1,1) and (1,3) of the potential matrix will remain unchanged (i.e., $\bar{v}_{1,2}^{3_1} = v_{1,1}^{3_1}$ and $\bar{v}_{1,2}^{1_3} = v_{1,1}^{1_3}$).

Since in each applied R&D facility the only n that agents expect to change is the one associated with the challenging x -class GPT, we can compare the productivities for any of the y facilities by simply comparing the $v_{y,z}^{(n-1)_x} (G_{(n-1)_x})_{t_{n_x}}$ that would be produced if the incumbent were left in place with the $\bar{v}_{y,z}^{n_x} (G_{n_x})_{t_{n_x}}$ that is expected to be produced if the challenger were adopted.¹⁵ This comparison is made in each of the Y applied R&D facilities at time $t = t_{n_x}$ so the test, stated generally for all applied R&D facilities, is:

$$(7) \quad \left[\bar{v}_{y,z}^{n_x} (G_{n_x})_{t_{n_x}} \right] \geq \left[v_{y,z}^{(n-1)_x} (G_{(n-1)_x})_{t_{n_x}} \right] \text{ for each } y \in [1, Y].$$

If the test is passed, the new GPT is adopted in facility y .

If none of the y applied R&D facilities adopts the GPT, it is returned to its pure knowledge industry. The indexes t_{n_x} and n_x are incremented back to their previous values—it is as if the favourable drawing of $I_t > I^*$ had not occurred. Pure research then continues to improve the new GPT and it is reconsidered every period until it is adopted or superseded in each applied R&D facility by a newer, superior version in its class.

The evolving efficiency with which the GPT delivers its services is shown in equation (8) below, which formalizes the efficiency curve discussed verbally in LCB Chapter 13 page 434.¹⁶

$$(8) \quad (G_{n_x})_t = (G_{(n-1)_x})_{(t-1)_{n_x}} + \left(\frac{e^{t+g(t-t_{n_x})}}{1+e^{t+g(t-t_{n_x})}} \right) \left(\Omega_{t_{n_x}}^x - (G_{(n-1)_x})_{(t-1)_{n_x}} \right).$$

The equation shows the efficiency of the GPT, $(G_{n_x})_t$, increasing logistically as the full potential of the GPT is slowly realized. t_{n_x} is the invention date of the version n_x , of the class- x GPT, $\Omega_{t_{n_x}}^x$ is the full *potential* productivity of the new version of GPT x ,

¹⁵ If the \bar{n} s attached to the other GPTs operating in applied R&D facility, y , were also expected to diverge from those in the appropriate column of the operative matrix, we would have to calculate the complete value of the a_t^y term from (2) as it would be under the existing GPTs and the operative set of n s and then compare it with what that a term would be given the new column vector of expected \bar{n} s.

¹⁶ Because there was only one GPT in existence at any one time in the models in LCB, we there interpreted equation (8) to represent both the efficiency and the applications curves, calling it a “logistic diffusion equation”

$(G_{(n-1)_x})_{t_{(n-1)_x}}$ is the *actual* productivity of the version that it replaced, evaluated at the time at which that earlier version was last used, $t_{(n-1)_x}$ and \mathbf{g} and \mathbf{t} are calibration parameters that control the rate of diffusion.¹⁷ The evolution of efficiency proceeds as follows. Initially, since $t_{n_x} = t$ (and because γ is very small, 0.07 in our simulations), the value of the efficiency coefficient is close to zero so that the initial productivity of the challenging GPT is close to that of the incumbent. As t increases over time the value of the efficiency coefficient approaches unity so that the GPT's productivity approaches its full potential.

In addition to modeling uncertainty sources 6 (iii) and 6 (iv) which we discussed above, the interaction of the G and the γ terms model uncertainty about when a new GPT will begin to be replaced by a challenger (source 6 (vi)), how long it will take to displace the incumbent GPT (source (vii)), and whether a GPT will be completely displaced (source 6 (viii)). Thus, they model the many aspects of the general observation that the applied potential of a GPT cannot be precisely predicted when it is originally being developed.

In the subsequent periods, the test in equation (7) is modified to note the productivity changes that occur over time:

$$(7') \quad \left[\bar{v}_{y,z}^{n_x} (G_{n_x})_t \right] \geq \left[v_{y,z}^{(n-1)_x} (G_{(n-1)_x})_t \right]$$

for each $y \in [1, Y]$ that has not yet adopted GPT G_{n_x} .¹⁸

In our model, the economy's GDP is the current period's output of the consumption and applied R&D sectors plus the cost of the composite resource used in the pure knowledge sector.¹⁹ The output of the applied R&D sector is an addition to the capital stock because that is what gets embodied in capital goods. In contrast, pure knowledge is an intermediate input into the applied research sector, being of no use in producing GDP until it is turned into applied knowledge. It is not, therefore, a part of the capital stock. Thus for our purposes, the stock of applied knowledge is the capital stock while and the flow of applied knowledge is gross investment.

Resource Allocation

As we have already noted, in the pure knowledge sector the government pays for and allocates a fixed amount of the generic resource, R , to each of the pure knowledge producing labs. Producers in the applied R&D and consumption sectors maximize their

¹⁷ Agents in our model do not know the parameter values of the logistic function and these can be either fixed or determined randomly in the model each time a new GPT arrives. When determined randomly, they model uncertainty source 6 (v).

¹⁸ This comparison must be made in every time period, for every applied R&D facility that has not yet adopted the latest version of every class of GPT in existence. It must also be made for all of the versions of any given class of GPT that are more recent than the one currently being used by a given facility.

¹⁹ Because the pure knowledge sector only periodically produces a useful GPT, we adopt the standard national accounting convention of valuing the output of that sector at its input costs in each period.

profits each period taking prices as given.²⁰ The prices for output from the I consumption industries are derived from the maximization of an aggregate utility function which we assume is additively separable across the I consumption goods.

$$(9) \quad U = \sum_{i=1}^I (c^i)^{f^i} \quad \text{and} \quad f^i = f^{i'} = 1, i \neq i' \forall i, i' \in I$$

The simplifying assumption that the exponents on each type of consumption are all equal to unity can, of course, be modified for specific applications. Maximizing this utility function yields:

$$(10) \quad \frac{MU^{i=1}}{MU^{i \neq 1}} = \frac{P^{i=1}}{P^{i \neq 1}} = \frac{f^{i=1} (c^{i=1})^{f^{i=1}-1}}{f^{i \neq 1} (c^{i \neq 1})^{f^{i \neq 1}-1}}$$

Since $f^i = 1 \forall i \in I$ it follows that $P^{i=1} = P^{i \neq 1}$, i.e., the relative prices of all consumptions goods are unity.

We assume a competitive equilibrium in the market for the composite resource. This implies that it earns the same wage, w , regardless of where it is allocated.

Each consumption industry maximizes its profits taking the price of its consumption output, P^i , and the prices of its inputs, composite resource, w , and applied knowledge, P^y , as given. Profits are expressed as:

$$(11) \quad \mathbf{p}^i = P^i c^i - w r^i - P^y A^y$$

Profit maximization yields the following FOCs in each of the I consumption industries:

$$(12) \quad \begin{aligned} P^i mpr^i - w &= 0 \\ P^i mpA^y - P^y &= 0 \end{aligned}$$

where mp represents marginal product. From the first FOC, the assumption the $P^i = 1$, and the definition of the production function for industry i we get:

$$(13) \quad r^{i*} = \left[\frac{\mathbf{a}_{Y+1}}{w} (\mathbf{m}A^y)^{a_y} \right]^{\frac{1}{1-a_{Y+1}}},$$

which is the reduced form expression for the demand for the composite resource in each consumption industry, i .

From the combination of both FOCs from the profit function for consumption industry i and the definition of the production function we get:

²⁰ We suppress time subscripts in equations (9) through (16) because agents are not foresighted and are consequently performing a static maximization in each period.

$$\frac{w}{P^y} = \frac{\mathbf{a}_{Y+1} A^y}{\mathbf{a}_y r^i}$$

which implies:

$$(14) \quad P^{y*} = \frac{\mathbf{a}_y w}{\mathbf{a}_{Y+1} A^y} \left[\frac{\mathbf{a}_{Y+1}}{w} (\mathbf{m} A^y)^{\mathbf{a}_y} \right]^{\frac{1}{1-\mathbf{a}_{Y+1}}}$$

Each applied R&D facility maximizes profits taking the price of its applied knowledge output, P^y , and the composite resource, w , as given. The pure knowledge input in the form the currently adopted set of X GPTs is provided freely to the applied R&D facilities by the government financed labs. Profits are expressed as:

$$(15) \quad \mathbf{p}^y = P^y a^y - w r^y$$

Maximization of the profit function and algebraic manipulation yields the following FOC:

$$P^y m p r^y - w = 0$$

The demand for the composite resource from each of the Y applied R&D facilities is thus:

$$(16) \quad r^{y*} = \left[\mathbf{b}_{X+1} \left[\prod_{x=1}^X (\mathbf{n}_{y,z}^{n_x} (G_{n_x})_t)^{b_x} \right]^{\frac{1}{X}} \frac{P^{y*}}{w} \right]^{\frac{1}{1-\mathbf{b}_{X+1}}}$$

With these resource demand equations we now have a complete description of the allocation of the composite resource across the three sectors.

IV THE MODEL SIMULATED

To simulate our model, we first make some simplifying assumptions, all of which can be relaxed for specific applications. We restrict the model to three industries within the consumption sector, three facilities in the applied R&D sector and three labs in the pure knowledge sector ($I = Y = X = 3$). We choose $\mathbf{a}_{Y+1} = \mathbf{b}_{X+1} = \mathbf{s}_{Y+1}$ and $\mathbf{a}_y = \mathbf{b}_x = \mathbf{s}_y$ to impose symmetry across sectors and specific activities within sectors (i.e., industries, facilities and labs). To simplify the utility function and make relative prices unity we set $\mathbf{f}_i = 1 \quad \forall i \in [1, I]$.

We choose values of the parameters and initial conditions with the overall objective that the model will replicate the accepted stylized facts of economic growth. We set the values of most of the initial conditions at unity because none of the long run characteristics of the model are influenced by initial conditions.²¹ Some values are chosen

²¹ The initial value of $t_{n_x} = 2$ is chosen because we have lagged variables indexed on it and MatLab does not allow zero as an index value.

to ensure consistency with observed data in the following ways:

$\mathbf{a}_{Y+1} = \mathbf{b}_{X+1} = \mathbf{s}_{Y+1} = 0.3$ ensures diminishing returns to the composite resource in all lines of activity; $\mathbf{e} = \mathbf{d} = 0.025$ produces an average annual growth rate between 1.5% and 2%;²² $I^* = 0.66$ allows a GPT within each class to arrive on average every 35 years, but with a large variance in arrival dates. We choose $\mathbf{g} = 0.07$ and $\mathbf{t} = -7$ to so that 90% of a GPT's potential is translated into its actual efficiency occurs over the first 120 years of its life. We choose $\mathbf{m} = 0.95$ in order to set the income shares of the labour and capital (physical and human) at approximately 0.3 and 0.7.²³ We set $\mathbf{a}_y = \mathbf{b}_x = \mathbf{s}_y = 1$ to ensure that knowledge has constant returns.

The following table gives the parameter values and initial conditions used to simulate the results of the multi-GPT model as reported in the text and shown in the figures.

TABLE 2
NUMERICAL SIMULATION OF THE MULTI-GPT MODEL

Parameter values:			
$\mathbf{a}_y = 1 \quad \forall i \in [1, Y]$	$\mathbf{a}_{Y+1} = 0.3$	$\mathbf{b}_x = 1 \quad \forall x \in [1, X]$	$\mathbf{b}_{X+1} = 0.3$
$\mathbf{s}_y = 1 \quad \forall y \in [1, Y]$	$\sigma_{Y+1} = 0.3$	$\mathbf{g} = 0.07$	$\mathbf{t} = -7$
$\mathbf{f}_i = 1 \quad \forall i \in [1, I]$	$\mathbf{e} = 0.025$	$\mathbf{d} = 0.025$	$\mathbf{m} = 0.95$
$I = 3$	$Y = 3$	$X = 3$	$I^* = 0.66$
$R = \sum_{i=1}^I r_t^i + \sum_{y=1}^Y r_t^y + \sum_{x=1}^X r_t^x = 1000$			
Initial conditions:			
$(G_{n_x})_0 = 1 \quad \forall x \in (1, X)$	$A_0^y = 1 \quad \forall y \in [1, Y]$	$n_x = 1$	$\mathbf{q}_0^x = 1$
$t_{n_x} = 2$	$P_0^i = 1 \quad \forall i \in [1, I]$		$d_{y,x} = 1$
$w_0 = 1$			

The set of \mathbf{q}^x are random variables distributed uniformly with support $[0.9, 1.1]$, mean 1, and variance $(0.4)^2/12$, which sets modest bounds on the uncertainty concerning the productivity of pure research. The I 's are derived from beta distributions, where each

²² We choose this range based on Angus Madison's historical data set (see Madison's web pages <http://www.ggdc.net/maddison/> for the complete data set). We calculate the average annual growth rate of GDP per person from 1870 to 2003 for the USA to be 1.86% and for Canada to be 1.96%.

²³ μ can alter shares because it alters the marginal product of labor in consumption, and labor is paid the same wage everywhere.

distribution is defined as $\text{beta}(x | \mathbf{y}, \mathbf{h}) = \frac{x^{(\mathbf{y}-1)} x^{(\mathbf{h}-1)}}{\text{Beta}(\mathbf{y}, \mathbf{h})}$ with support $[0,1]$, mean $(\mathbf{y}/(\mathbf{y}+\mathbf{h}))$

and variance $\frac{\mathbf{y}\mathbf{h}}{(\mathbf{y}+\mathbf{h})^2(\mathbf{y}+\mathbf{h}+1)}$. $\text{Beta}(\mathbf{y}, \mathbf{h})$ is the Beta function, and \mathbf{y} and \mathbf{h} are

parameters which take on positive integer values. We choose $\mathbf{y} = 5$ and $\mathbf{h} = 10$.

Next we need to determine the ρ 's in each challenger's potential matrix. They are calculated as the ρ 's in the current operative matrix modified by the addition of a random variable, \mathbf{r} , drawn from a uniform distribution with support $[-0.05, 0.05]$. This ensures that the ability of a type- x challenging GPT to cooperate with the other types of incumbent GPTs used in a given research facility are related to the ability of the type- x incumbent to do the same thing and the support for \mathbf{r} is chosen so that deviations from operative to potential ρ s are not large. For the example in Table 1, the potential ρ s associated with a challenger of version $n+1$ from class $x = 2$ are derived from the incumbent's ρ s as follows:

$$(16) \quad \mathbf{n}_{y,2}^{(n+1)_2} = \begin{cases} \mathbf{n}_{y,2}^{n_2} + \mathbf{r} & \text{if } (\mathbf{n}_{y,2}^{n_2} + \mathbf{r}) \in [0.5, 1.5] \\ 0.5 & \text{if } \mathbf{n}_{y,2}^{n_2} + \mathbf{r} < 0.5 \\ 1.5 & \text{if } \mathbf{n}_{y,2}^{n_2} + \mathbf{r} > 1.5 \end{cases} \quad y \in [1, 3]$$

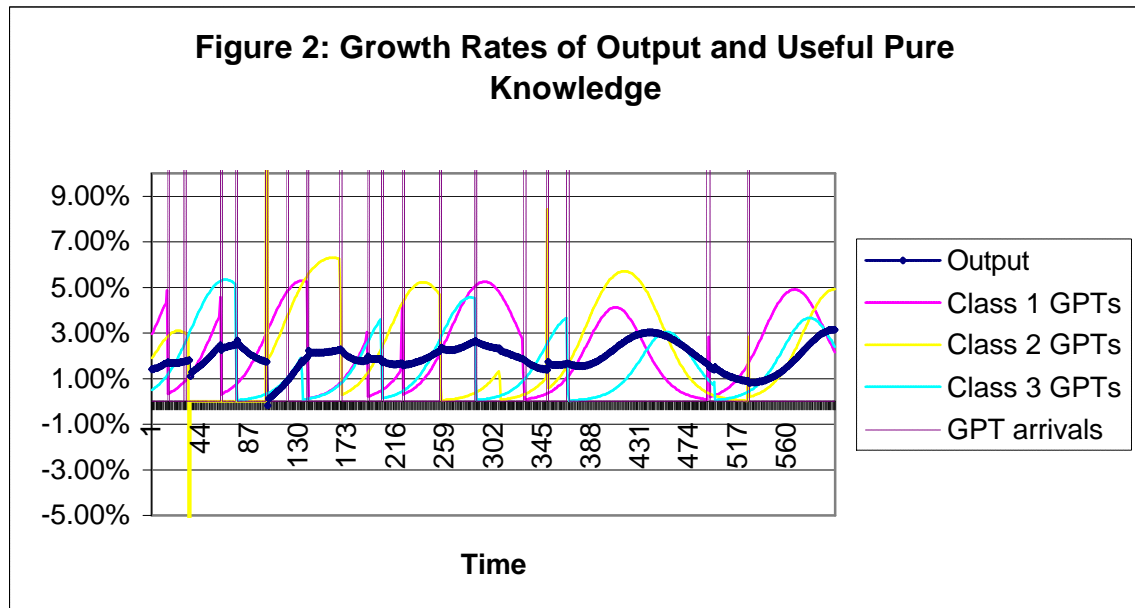
V THE MODEL APPLIED

To demonstrate some important properties of our model and illustrate its applicability to practical issues, we provide three illustrations. In the appendix, we show that, despite its randomness, path dependence and non-stationarity in levels and/or rates, the model is consistent with the generally accepted stylized facts of economic growth as laid out by Kaldor (1961) and Easterly and Levine (2001). In the text, we first consider the relation between individual GPTs and the whole economy and then look at an illustrative policy issue.

Because all past models allow only one GPT to exist at any one time, the performance of the whole economy closely mirrors that of the existing GPT. This has led some researchers to look to the economy's performance to deduce something about the arrival and impact of an alleged GPT. We might suspect, however, that the relation between an individual GPT and the whole economy that was present in a one-GPT-at-a-time model would not show up when several GPTs all at different stages in their evolution exist at any one time. Figure 2 illustrates that this suspicion is correct in our model. The heavy blue line is the growth rate of aggregate output (as shown in more detail in Figure A1 in appendix) while the lighter lines are the growth rates of the efficiency of each class of GPT as determined from equation (8) in the simulation.

The figure shows both that the arrival and evolution of any one GPT does not typically determine the pattern of growth of aggregate output and that the growth of aggregate output cannot be used to infer the arrival and evolution of individual GPTs in a multi-GPT world. The spikes in the growth rate do not just signal the arrival of a new GPT. They occur whenever a new GPT is adopted by a particular applied facility, which may occur long after the original arrival of a new GPT. Sometimes the spikes are

negative because of a bad realization of the θ s for GPTs of other classes that cooperate with the new GPT. Eventually however the new GPT does pay off in enhanced productivity.²⁴



We now illustrate the potential of the model to address policy issues by considering the appropriate mix of public funding to support pure and applied research. We make a number of simulations in which pure research is solely financed by the government while applied R&D is financed by a mix of government and private funds. In total, the government allocates 3.4 % of GDP to knowledge production.²⁵ In the first simulation, all of the government support goes to pure knowledge production (Policy 1). In the second, after 20 iterations, the government reallocates its funds to split them evenly between pure and applied R&D (Policy 2). In the third, again after 20 iterations, the government allocates all funding to applied research (Policy 3). To make these three simulations display solely the effects of the different research allocation policies, we ran the first simulation then imposed the realized random variables from that simulation on the next two.

²⁴ The negative shock followed by a productivity gain is similar to what we seen in the 1970s-90s with the ICT revolution.

²⁵ We choose 3.4% because the United States allocates about 2.63 % of total GDP to R&D and 2.3% of GDP to post-secondary education and research making 4.93% in total. But since this ignores the significant fraction of total government expenditure that goes to R&D, particularly from the Department of Defense, this number would need to be significantly increased for practical applications. The upper limit would be the total public expenditure (not including transfers) which is 19.1% of GDP.

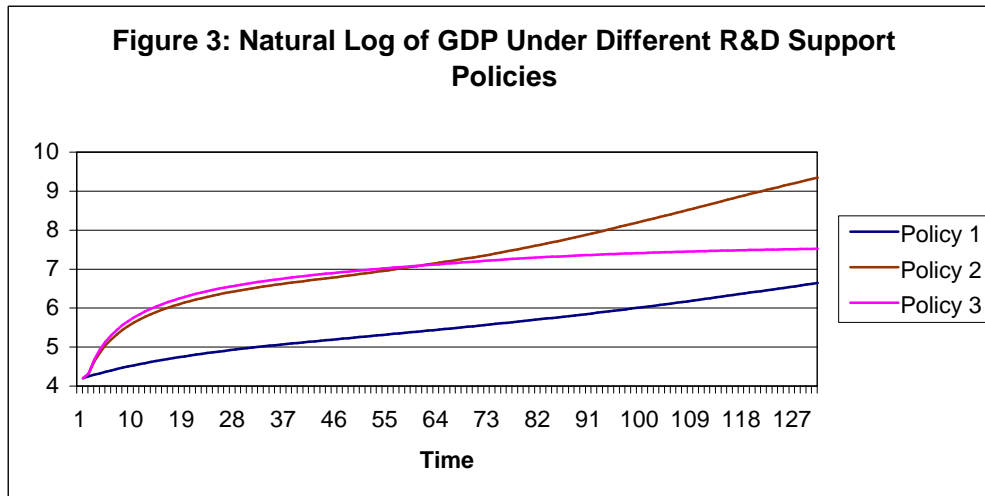


Figure 3 gives the natural logs of the GDP paths, starting at the date of these alternative policy changes. It shows that there is a large gain over any time period in having the government support some applied R&D (Policy 2) rather than solely supporting pure research (Policy 1). The best mix would have to be discovered iteratively and some experimentation shows that it is closer to 80% for applied R&D, rather than the 50% shown in the figure. The government must not, however, go too far. The figure shows that there is an early gain to allocating all public support to applied R&D (Policy 2) — provided, of course, that some amount of pure knowledge has been accumulated and that existing GPTs have some unexploited potential. This gain is eventually more than offset by the loss from not maintaining support for the pure knowledge production that creates new GPTs. As the figure shows, Policy 3 ceases to out-perform Policy 2 after about 50 iterations and, since it eventually leads to a constant income (zero growth), it is eventually also out-performed by Policy 1.

Although this analysis is crude, the qualitative results suggest some interesting possibilities for further study in more sophisticated, context-specific models.

1. Given that the public sector is financing pure research, the private sector may under-produce applied R&D making some mix of private and publicly supported applied R&D preferable.²⁶
2. The highest possible rate of growth can be attained, and sustained for some time, by diverting all research resources to applied R&D. However, the absence of new GPTs created by pure research is sooner or later felt in a falling growth rate that

²⁶ The reason for point 1 is that in our model private agents allocate resources to applied R&D based on the effect of such research on the near-future productivity of the consumption goods industries. If private sector agents looked further into the future, there would be an incentive to allocate more to current R&D. However as long as they only look a few periods ahead when calculating their expected returns (as is often alleged) the public return from such R&D will exceed the private return, providing a case for some diversion of public funds to encourage private R&D (as governments do in many countries by policies such as tax relief or direct subsidies for R&D).

eventually reaches zero, making this the worst allocation policy in the very long term.

Further extensions of the model and consideration of the many policy issues it can be used to study must await further papers.

APPENDIX

MEETING THE STYLIZED FACTS OF ECONOMIC GROWTH

In this appendix, we show that our model can duplicate the relevant, generally accepted stylized facts of economic growth as laid out by Kaldor (1961) and Easterly and Levine (2001)

I. KALDOR'S FACTS

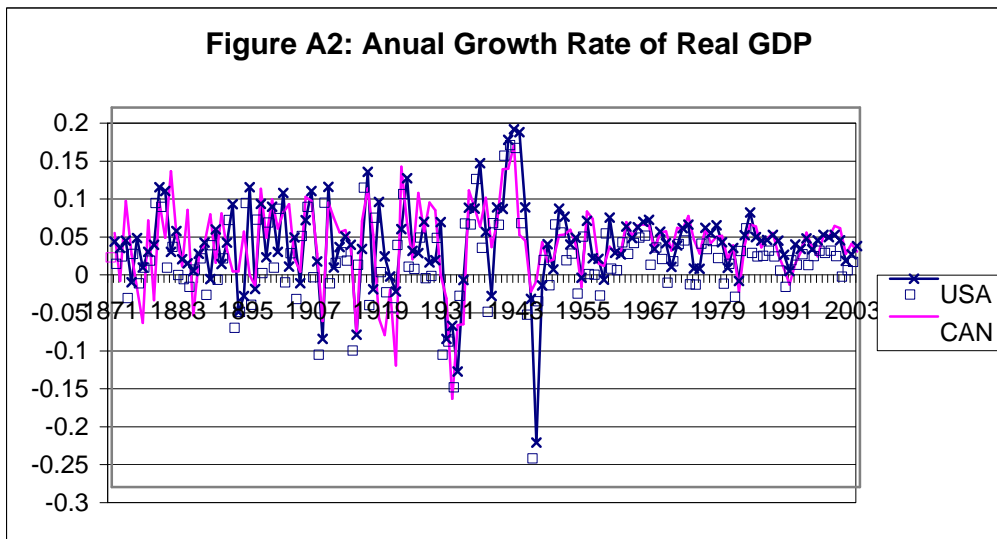
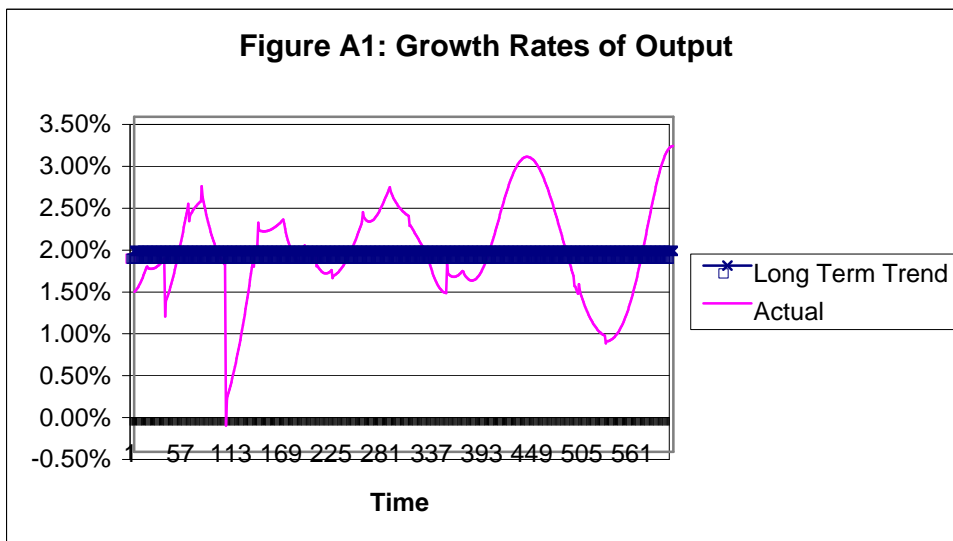
Kaldor's (1961: 178-79) stylized facts were as follows.

1. "The continued growth in the aggregate volume of production and in productivity of labour at a steady trend rate; no recorded tendency for a *falling* rate of growth of productivity." The only way to make this consistent with fact 6 below is to assume that 'steady trend' means a sustained growth on average over the decades, not that growth rates have been relatively constant year by year or even decade by decade.
2. There has been "[a] continued increase in the amount of capital per worker whatever statistical measure of 'capital' is chosen in this connection."
3. There has been "[a] steady rate of profit on capital, at least in the 'developed' capitalist societies; this rate of capital being substantially higher than the 'pure' long-term rate of interest as shown by the yield of gilt-edged bonds."
4. There have been "[s]teady capital-output ratios over long periods; at least there are no long term trends either rising or falling, if differences in the degree of utilization of capacity are allowed for."
5. There has been "[a] high correlation between the share of profits in income and the share of investment in output; a steady share of profits (and of wages) in societies and/or in periods in which the investment coefficient (the share of investment in output) is constant."
6. "Finally, there are appreciable differences in the *rate* of growth of labour productivity and of total output in different societies, the range of variation (in the fast-growing economies) being of the order of 2-5 per cent. These are associated with corresponding variations in the investment coefficient, and the profit share, but the above propositions concerning the constancy of relative shares and of the capital-output ratio are applicable to countries with differing rates of growth."

To reproduce the first of Kaldor's stylized facts we ran the base line simulation several times to ensure that we did not report a case where outlying values of the random variables had been realized. The average growth rate of output over several simulations with 600 iterations per simulation and with a new version of a GPT in each class arriving on average every 35 years, was 1.72% with a maximum growth rate in one simulation of 2.31% and a minimum in another simulation of 1.63%. Because labour is modeled as a

component of our generic ‘resource’ and the resource endowment is constant, the sustained growth rate implies a sustained rate of growth in labour productivity. An alternative way to observe the growth of labour productivity is through the natural log of the wage rate for resources, which represents the value of the marginal product of resources, mainly labour and land in our model. This figure increases by about 7 fold over 600 iterations in a typical simulation.

Figure A1 shows our model’s growth performance for a typical iteration, in which the long term average growth rate is 1.9%. The actual rate fluctuates around this average quite dramatically and there is no discernable pattern of convergence between the long run average and the actual growth rate whose variance is 0.0018.



Source: A. Madison <http://www.ggd.net/maddison/>

Are the fluctuations in Figure A1 unrealistically large? We answer that they are less than those found in the real data. Growth rate spikes occur in both the simulated data of Figure A1 and the real data shown in Figure A2, Also the variance of

the simulated growth series is not as large as that for the actual growth rates of GDP for Canada (0.00264) and the US (0.0029) calculated from Madison's data shown in the figure. The key point is that period by period growth rates are quite volatile for both the real and the simulated data.²⁷

Our capital stock, defined in the main paper as the stock of embodied applied knowledge, exhibits a sustained growth rate that is equal to the growth rate of output (i.e., on average over the set of simulation conducted it is 1.72%). Therefore, given a fixed pool of labour, capital per worker is rising throughout any simulation, thus meeting Kaldor's second stylized fact..

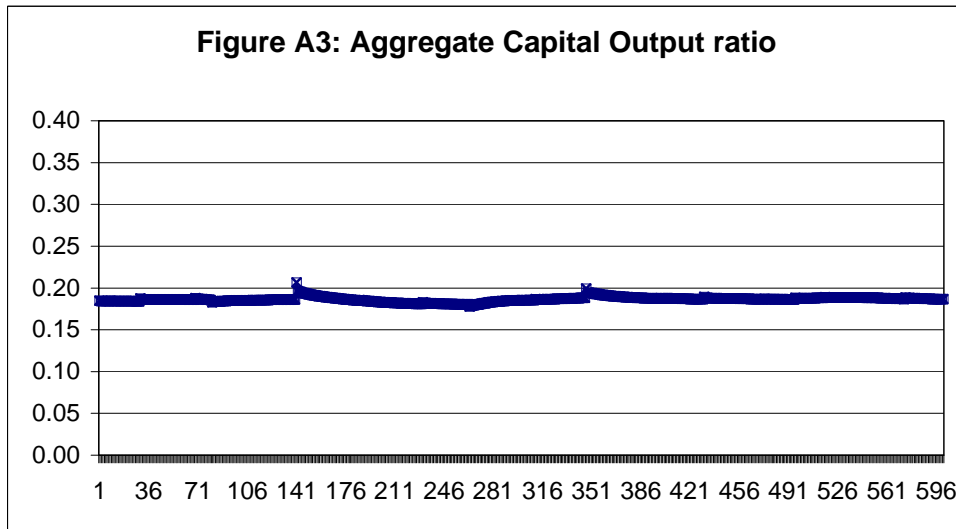
Because the capital stock is growing as a consequence of the continued allocation of resources to the production of applied knowledge and these resources have an alternative use in producing consumption every period, capital earns a positive rate of return in our model.²⁸ But because there are no guilt edged or other risk-free investments in our model, this rate of return cannot be compared with other endogenously generated rates.

Figure A3 shows that the capital output ratio $\frac{\sum_{y=1}^y A_t^y}{\left[\sum_{i=1}^I c_t^i + \sum_y^y a_t^y + w^* \sum_x^X r_t^x \right]}$ is relatively

constant over time, thus meeting Kaldor's fact 4. This result is not particularly surprising because the growth of the applied knowledge stock, which incorporates the pure knowledge stock, drives the growth of measured GDP in our model.

²⁷ Our lower volatility is because we do not model the impact on the growth rate of many exogenous shocks such as wars and weather.

²⁸ E. Böhm- Bawerck (1889) and K. Wicksell (1893) were among the first to argue that the payoff to creating intermediate capital paid for by deferring current consumption must be a return of future output sufficient to overcome the individual's rate of time preference.



The income shares for labour and capital (measured here as payments to resources

divided by output, $Labshare = \frac{w^* \left[\sum_{i=1}^I r_t^i + \sum_{y=1}^Y r_t^y + \sum_{x=1}^X r_t^x \right]}{\left[\sum_{i=1}^I c_t^i + \sum_y a_t^y + w^* \sum_x r_t^x \right]}$, and payments to applied R&D

knowledge stock, $Capshare = 1 - Labshare$) are stable over the 600 iteration time horizon of the simulation with the average capital income share at 0.68 and the average resources share at 0.32, thus meeting Kaldor's fifth stylized fact. These numbers are conditional on certain parameter values in the production function, which if changed will alter the division of income between capital and labour but will still result in the division being constant over the simulation. If we take into consideration that we have not distinguished between human and physical capital and guess that human capital is on average equally important as physical capital in the applications of pure knowledge then human and physical capital's share should each be 0.34. These numbers are close to those estimated by Gregory N. Mankiw, David Romer and David N. Weil (1992). However, ours is an endogenous growth model while they were estimating an exogenous model.²⁹

To meet Kaldor's sixth stylized fact, we ran a variety of simulations and let them represent different societies with different factor income shares, rates of investment, arrival dates of GPTs, and realizations of relations among them (the ?s), The differences in these variables caused the simulations to generate different rates of growth in total output and labour productivity.

²⁹ Some authors (see, for example, Greasley and Oxley (1996)) have empirically tested whether exogenous models of the sort that Mankiw, Romer and Weil estimated or endogenous models have more explanatory power in the available data. Most of the evidence indicates that there is no way to reject one or the other type of model.

II. EASTERLY AND LEVINE

We now consider Easterly and Levine's (2001) set of stylized facts, which are listed on page 177 of their article.

1. "The 'residual' (total factor productivity, TFP) rather than factor accumulation accounts for most of the income and growth differences across countries."
2. "Income diverges [across countries] over the long run."
3. "Factor accumulation is persistent while growth is not, and the growth path of countries exhibits remarkable variation."
4. "Economic activity is concentrated, with all factors of production flowing to the richest areas."³⁰
5. "National policies are closely associated with long-run economic growth rates." Since the record of government with respect to policies designed to influence technological change and economy growth is spotty, we take this stylized fact mean that such policies *can* affect growth.

With respect to E&L's first fact, we take their statement about the 'residual' to refer to the state of technology, although for reasons we have detailed elsewhere, and allude to below, changes in TFP do not typically measure technological change. To accommodate their fact we conduct the following experiment. We switch off most of the model's randomness by setting q and all of the v s equal to 1 for all time periods. The remaining source of randomness is the arrival dates of GPTs, which we keep the same across our simulations by letting them be determined by our I mechanism in the first run and then imposing that sequence of I s in the other runs. We ran three simulations in which the tax transfer system allocated respectively 100 units, 1 unit and 0 units of the resource to each of the three pure knowledge labs. We interpret the results as the behaviour of three different countries with different S&T policies. The average growth rates in the three simulations were respectively 1.74%, 1.13% and 0%. Thus the three countries have different growth rates which are accounted for by different rates of technical progress. In our model this progress has two sources, the break through inventions of new GPTs coming from the pure research labs and the complimentary innovations produced by the applied R&D facilities as they embody the GPT in new productive capital. So part of technological progress is measured as the continual accumulation of knowledge produced by the pure research labs and part by the growth in the capital stock that is produced by the applied R&D facilities and that embodies new technological knowledge.³¹

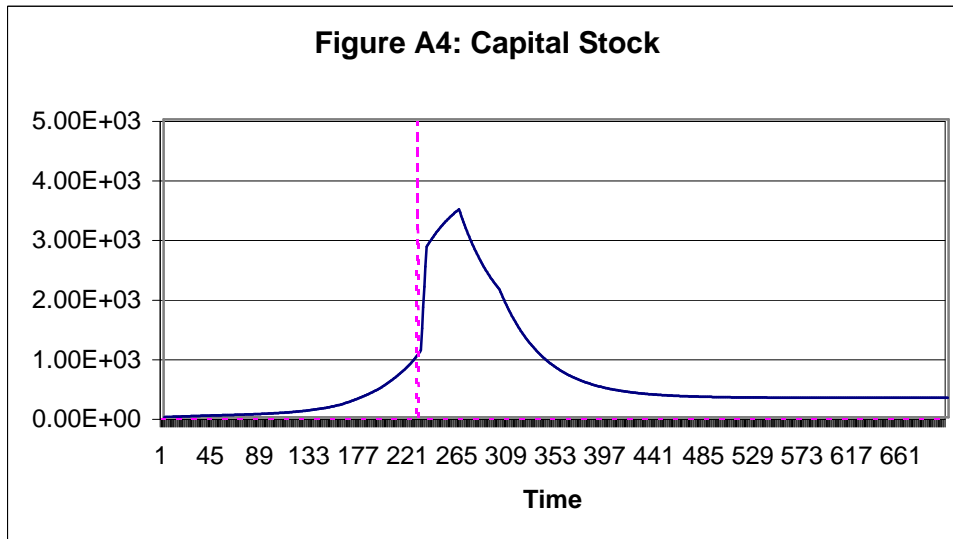
³⁰ This fact is contrary to the predictions of the Heckscher-Ohlin model, which predicts that mobile factors will migrate from countries where they are relatively plentiful to countries which are relatively scarce, thus producing two way flows of factors. The failure of this prediction suggests important externalities and scale effects

³¹ This implies that technological change cannot be measured by changes in TFP because the knowledge gets embodied in the capital stock and, therefore, does not appear as the residual of the growth of output after accounting for the growth of inputs. This point has a long history going back as far as Dale W. Jorgensen and Zvi Griliches (1967). We deal with it in detail in Lipsey and Carlaw (2004) and Lipsey, Carlaw and Bekar (2005: Appendix to Chapter 4). Note also that the critical driving force of growth is the

We next address stylized facts two and five area addressed both with the experiment just described and the one in the text illustrated in Figure 3 in which the government alters the division of its support between pure and applied research. In both of these cases the growth rates, and hence income levels, differ across countries, consistent with fact 2. Since these differences in growth rates were the result of different government policies in allocation resources to pure research, these results are also consistent with E&L's stylized fact five as long as we interpret it to say that government can *sometimes* influence growth rates by their policies — but not invariably because the record of actual national policies is spotty.

Fact 3 is accommodated by another experiment. At some point in time (iteration 200 in the simulation), we cut off all new pure research and all improvements in the efficiency of existing GPTs (the evolution of the G s described in equation (8) is halted so that the G in each applied R&D facility's production function is frozen at its current level). The stock of capital measured as accumulated applied knowledge is as shown on a natural scale in Figure A4. Although we cannot have capital accumulation with constant technology in our model, we do have accumulations of capital for a period of time with no new GPTs and no improvements in the efficiency of existing GPTs. In this case, the applied R&D facilities make do with the technologies they have but continue to develop new applied knowledge on this constant base. There is an immediate spurt in growth as resources formerly used in pure research reallocate to the applied R&D facilities and the consumption industries. But after that, the growth rate quickly slows, then becomes negative for a time as the stock of applied knowledge is eroded due to depreciation and eventually reaches zero as the stock of knowledge converges on its steady state value. Thus even in this model where there is some development of new applied knowledge, the growth rate reaches zero, which illustrates that pure capital accumulation with no new technological knowledge can persist for some time but would produce an even more rapid convergence of the growth rate to its zero equilibrium.

accumulating stock of pure knowledge that periodically results in new GPTs. This pure knowledge is an intermediate input that is used to create new kinds of capital and is only indirectly measured by changes in the capital stock.



We cannot deal with Easterly and Levine's fourth fact because, like all other existing GPT models, ours only handles one country at a time. We could set up a system of multiple jurisdictions and free moving factors of production but that is another research project.

Thus, our simple experiments meet all but one of the stylized growth facts. Some of these could also be met by allowing random variability to play a role. For example, a country may experience bad draws from all sources of variation in the model producing a lower growth experience.

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