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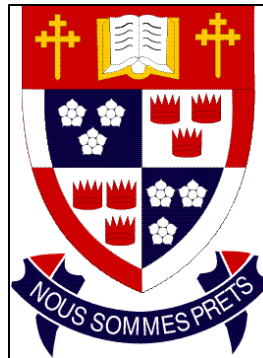
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**GTP-Driven, Endogenous
Growth with Uncorrelated
changes in TFP**

K. Carlaw
R. Lipsey

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SIMON FRASER UNIVERSITY

GPT-DRIVEN, ENDOGENOUS GROWTH WITH UNCORRELATED CHANGES IN TFP

by

Kenneth I. Carlaw

Lecturer, University of Canterbury, New Zealand

email: k.carlaw@econ.canterbury.ac.nz

and Richard G. Lipsey*

Emeritus Professor of Economics at Simon Fraser University

email: rlipsey@sfu.ca,

home page: <http://www.sfu.ca/~rlipsey>

ABSTRACT

We present a non-standard aggregate model in which sustained growth is driven by a succession of different general purpose technologies (GPTs). The model has diminishing returns to the factor inputs (the K of AK models), and endogenous, incremental technological changes in applied R&D, interrupted by the occasional introduction of new GPTs. The latter are developed by endogenous R&D, which has a payoff that is subject to uncertainty with respect to its magnitude and timing. Although the model is technically much simpler than those in Helpman (1998), it captures many of the complex inter-relationships involved in the growth process that could not be covered by those first generation GPT models. Calculations made on a number of different data sets simulated from the model reveal that TFP growth can easily miss-measure the rate of technological change.

Key Words: sustained growth, endogenous growth, general purpose technologies (GPT), total factor productivity (TFP), spillovers.

GPT-DRIVEN, ENDOGENOUS GROWTH WITH UNCORRELATED CHANGES IN TFP

Capitalist economies have produced continued growth in per capita income, at least since the end of the eighteenth century, but not at a constant rate. Growth rates that vary over the long term cannot be explained by theoretical models that produce either constant growth along a steady state or a balanced growth path or continually accelerating or decelerating growth rates. These include many of the models of equilibrium endogenous and exogenous growth, as well as real business cycle models. The latter produce cyclical patterns around a steady state or balanced growth equilibrium — patterns that are generated by exogenous stochastic shocks unrelated to endogenous choices of firms concerning their technology. Variable growth rates also present a problem for measurements of technological change based on changes in total factor productivity (TFP) since they typically rely on the same theoretical models. As a result, changes in TFP do not necessarily track changes in technology.

We use a succession of general purpose technologies (GPTs) as the key to explaining variable growth rates over the long term.¹ A study of the history of technological change shows that GPTs arrive at widely dispersed times and more than one is usually working through the system at any one time. The evolution of a GPT and its new derivative technologies create a set of spillovers, many of which neither fit the definition of technological externalities nor are caught by conventional measures of them. These spillovers permeate the economy, enabling many subsequent technological changes in the forms of new product, process and organizational technologies and improvements in existing ones. Each GPT brings its own research program whose time series of cumulative results typically follows its own logistic curve, rising slowly at first, then with increasing rapidity as each new innovation enables others, and finally slowing as opportunities to exploit the GPT become exhausted. Although some programs are richer than others, there is *no* reason to expect that successive GPTs will always either accelerate or decelerate growth on average over their lifetimes. But they do sustain it. As long as GPTs continue to arrive on the scene, growth will continue — sometimes faster and sometimes slower.²

This is the third in a series of three related papers in which we deal in one way or another with the type of non-stationary long term economic growth that is driven by a succession of GPTs. In the first paper, “The Measurement of Technological Change” (2003, forthcoming), Lipsey and Carlaw argue that TFP does not measure technological change. Instead, it is an imperfect measure of a subset of the spillovers that are produced by growth-creating technical change. In the second paper, “Externalities versus Technological Complementarities” Carlaw and Lipsey (2003, forthcoming), distinguish *technological complementarities*, which are the full set of spillovers associated with technological change, from *technological externalities*, which are the subset that TFP calculations measure (albeit imperfectly). In this, the third paper in the series, we seek to improve on existing GPT models of the sort given in Helpman (1998) and, in doing so, to illustrate within the confines of a formal model, some of the more general arguments given in the other two papers. We do this, first, by developing a model in which endogenously generated GPTs create spillovers that cause sustained growth whose average rate varies from one GPT to another and, second, by demonstrating that in our model changes in TFP do not track changes in technology. Our model does two main things. First, it removes some,

although not all, of the set of unrealistic simplifying assumptions that were needed to make the first generation of formal GPT models operational. Second, the analysis of various results for the measurement of TFP in our section 3 goes a little way towards answering Prescott's (1998) call for a theory of TFP.

1. INTRODUCTION.

1.1 Main Features of Our model

We first outline the main features of our three-sector model of endogenous technological change and economic growth in which GPTs are created by maximizing agents under conditions of uncertainty. In it, resources are allocated among three sectors as if perfect competition prevailed. The sectors are (1) "consumption," which is all consumer's goods and services and which we assume can be aggregated into the single output index for each time period (2) R&D to develop applications of the GPTs to specific purposes, called "applied-R&D" and (3) R&D that helps to develop GPTs, called "fundamental research." Each of the three sectors is represented by a single production function.

The production system explicitly incorporates technological spillovers such that GPTs rejuvenate the growth process by making applied-R&D more productive while the productivity of fundamental research is increased over time by both applied-R&D and fundamental research. These relations are intended to mirror the types of empirical observations discussed in the second paper in this series (Carlaw and Lipsey (2003)).

We introduce uncertainty in three ways. First, the amount of new knowledge generated by resources devoted to fundamental research in every period is uncertain. Second, the time between the arrival of successive GPTs is of uncertain duration (but typically long). Third, the effect that the GPT has on productivity in the R&D sector when it does arrive is uncertain. These characteristics are designed to conform with the empirical evidence that the search for new fundamental technologies is an uncertain venture in three senses that (i) no one can know exactly how much knowledge will be generated by a given amount of effort (ii) success occurs only occasionally and (iii) the applied potential of GPTs varies from one GPT to another.

Agents maximize expected output in each period in a recursive fashion,³ taking the marginal products in each sector as given. However, the marginal products in each sector change in the next period as a result of the allocation of resources in this period. This implies that the model does not employ a stationary dynamic equilibrium concept. There is a transitional competitive equilibrium in every time period, given the marginal productivities of inputs in each sector. But because of technological advance and the nature of the spillovers, the marginal products change from one period to the next, making the competitive equilibrium outcome different in each period. Thus the economy never settles into a stationary equilibrium (neither a steady state nor a balanced growth path). Although growth never stops, a very productive new GPT can accelerate the average growth rate over its lifetime while a less productive new GPT can slow it.

Our model has the following key behavioral characteristics. GPTs arrive at randomly determined times but with a productivity that is determined by the amount of pure research knowledge that has been endogenously generated since the last GPT and a random component. The average growth rate of income over the lifetime of each successive GPT is variable. Although GPTs arrive randomly, the average growth rate

over long periods of time in which several GPTs succeed each other is determined by the accumulated amount of pure knowledge. This is determined partly endogenously by the allocation of resources to pure research and partly exogenously by random factors affecting the productivity of those resources.

Our formulation of knowledge production — one process for pure knowledge and one for applied knowledge — raises important issues when we come to calculate total factor productivity. New knowledge affects the system's production functions and it matters whether this knowledge is treated as *an input into*, or as *a shift of*, those production functions. We model new knowledge as shifting the production function, but demonstrate in section IV the effects on TFP measures if knowledge is treated as an input.

1.2 Comparison with other models

Next we compare our model with four closely related models: the first three, Aghion and Howitt (1998a), Helpman and Trajtenberg (1998a) and Helpman and Trajtenberg (1998b) are the models of GPTs that are found in Helpman (1998)⁴ while the fourth, Aghion and Howitt (1998b), is a model of endogenous technological change with creative destruction. (We refer to these four as “the comparison set,” to the first three as “the GPT models” and to the fourth as “A&H(T).”)

While all the GPT models are concerned with the economy's behavior over the lifetime of one GPT, particularly with GPT induced slowdowns in productivity growth, our model is concerned with long term economic growth over the life times of many successive GPTs.

In our model, the fully employed resources are allocated between the three sectors, one of which is generating pure knowledge that contributes to the emergence of new GPTs. Three of the models in the comparison set have only two sectors, with resources allocated between current production and R&D to develop components that are complementary with each new technology (GPTs in both Helpman and Trajtenberg papers). These arrive exogenously in Helpman and Trajtenberg (1998a and 1998b) but the arrival rate is partly endogenous in A&H (T). The fourth, Aghion and Howitt (1998a), has these two sectors plus a third sector that contains a set of resources that are confined to it. These special resources produce “templates” that are needed before a new GPT can be useful. They are unemployed until the exogenous arrival of a new GPT; they are then employed for a while but return to unemployment while the new GPT is in use.

We assume “as if” perfect competition, to avoid the existence of pure profits that would influence our TFP calculations. All the other models assume monopolistic competition in at least one sector. This gives them rents that are associated with property rights over the components and that are destroyed by new GPTs (or new technologies in A&H (T)).

Our first source of randomness, uncertainty about the amount of pure knowledge generated by a given amount of resources, has no counterpart in the models of the comparison set because none of these allow for the generation of knowledge that contributes to the creation of new GPTs. The second source of randomness in our model comes from the arrival rate of GPTs. Helpman and Trajtenberg (1998a) and (1998b) have GPTs arriving exogenously at predetermined intervals while Aghion and Howitt (1998a) have GPTs arriving exogenously with a probability so small that

they only worry about the history of a single GPT. A&H (T) have technologies arriving at a rate determined by a Poisson arrival process that is endogenously influenced by an instantaneous flow of labor services. Our third source of randomness makes the productivity of different GPTs vary even though they may contain the same amount of accumulated knowledge. In contrast, all other GPT models have identical GPTs. In A&H (T), the productivity and timing of new technologies is endogenously determined and, therefore, constant in their stationary equilibrium.

In our model, the amount of new applied knowledge is determined by the amount of resources allocated to applied R&D. This is also true of all the models in the comparison set with the exception of A&H (1998a). In that model there is a minimum amount of applied knowledge that must be accumulated after a GPT appears but before it can be used productively and the accumulation of this knowledge is subject to a random process.

In all of the models, including ours, sustained growth requires the emergence of new technologies and the existence of spillovers in one form or another.

Some of the most important differences between our model and those in the comparison set follow from different equilibrium concepts. Our agents cannot foresee the consequence of the spillovers arising from their present resource allocation decisions. Thus, our model does not have a stationary equilibrium. This allows us to focus on the historical, path dependent and variable pattern of growth. Our innovation rate is variable and the pattern of knowledge accumulation is a path dependent process. The historical pattern of knowledge accumulation depends on the realizations of the various random variables in each period and the allocations of resources that occur given the accumulated experience of all previous periods. The amount of resources devoted to fundamental research is, however, endogenous and over the "centuries" this determines the total impact of GPTs, even though the precise arrival of each GPT is random. So over the long term the amount of GPT-driven growth (i.e., its cumulated productivity effect) is largely endogenous even though, arrivals are random and the shorter term variations in the time path are random.

In all the models of the comparison set, agents are assumed to be able to foresee the whole course of the evolution of a new GPT the moment it arrives. This allows them to maximize over the life time of the GPT or new technology, which produces a stationary equilibrium derived from their infinite horizon utility maximization. Even in A&H (T) where there is randomness in the arrival rate of new technologies, innovation is at a constant rate in equilibrium. This is because their innovation arrival rate is derived from the expected value of the Poisson distribution with a parameter determined by the equilibrium flow of labor services into research.

Since most GPTs arrive as crude, single-purpose technologies and take decades to evolve into fully fledged GPTs, our assumption of no foresight seems closer to what we observe than the assumption of perfect foresight commonly made in other GPT models. Nonetheless, faced with our assumption, one might wonder if, by learning over successive GPTs, agents might eventually be able to anticipate the course of each new one. But the facts suggest otherwise. Since GPTs are technologically distinct from each other, the history of past GPTs provides little quantitative evidence about how new ones will behave. For example, knowing how the steam engine affected the economy over the several hundred years of its evolution would tell agents virtually nothing about the evolutionary path to be expected for electricity at the time when the dynamo was invented in 1887.⁵

2. THE FORMAL MODELS

In all of our models we use the following assumptions concerning inputs and consumption.

Inputs: First, the standard inputs are expressed as a single aggregate, R , which is allocated among the three sectors:

$$R_t = r_{c,t} + r_{a,t} + r_{g,t} \quad [1]$$

where $r_{c,t}$, $r_{a,t}$ and $r_{g,t}$ are respectively the factors devoted to the production of consumption, applied R&D, and pure research. Second, the r s enter the production function of each sector with a positive exponent of less than unity giving rise to decreasing returns to these inputs. We justify this assumption by observing that some of the variables that affect production are more or less fixed (e.g., population in industrialized countries and natural resources everywhere), causing those that can be accumulated, our r s, to be subject to decreasing returns.⁶ Third, we assume that R is an exogenously determined parameter. We could allow R to be accumulated endogenously and then, ceteris paribus, it would increase up to some steady state value. So, allowing for saving and accumulation in our framework produces the result that factor accumulation with labor constant, and without technological change, runs into decreasing returns — the same result as in the Solow model, with capital accumulating while labor and technology are held constant. Since this would not alter any of our conclusions, we do not model factor accumulation.

Sector 1, Consumption: Consumer goods are produced by an aggregate production function

$$c_t = \mu A_t r_{c,t}^\alpha \quad [2]$$

where $\alpha \in (0,1)$, μ is a parameter that is needed in our three sector model, and A_t is a productivity parameter that is determined by the accumulated knowledge produced in the applied R&D sector. Increases in A_t cause increases in the marginal productivity of resources in consumer goods. (Although we treat knowledge here and in equations [3] and [5] as shifts in the production function, there is nothing in the mathematics that prevents us from interpreting it as an input in the production function.) If applied R&D knowledge were treated as an input, the production function for consumption would exhibit increasing returns to scale and constant returns to knowledge.

2.1 A Two Sector-Model

To illustrate how growth is sustained and to provide a benchmark for our complete model, we first introduce a model that contains only two sectors, consumption and applied R&D. We thus assume that $r_{g,t} = 0$ in [1] and $\mu = 1$ in [2].

Sector 2, Applied R&D: The second sector is applied R&D. This sector creates capital for the consumer goods sector with a production function

$$\begin{aligned} a_t &= v G_t^a r_{a,t}^\beta \\ A_t &= a_t + (1 - \varepsilon) A_{t-1} \end{aligned} \quad [3]$$

where $\beta \in (0,1)$, v is a parameter used in the simulation to calibrate the model, ε is the rate of obsolescence that applies to the stock of R&D-produced knowledge and G_t^a is a productivity parameter that is determined by the accumulated knowledge

produced in the GPT sector and assumed exogenous in the two sector model. Increases in G_t^a raise the marginal productivity of resources in applied R&D and this in turn affects the marginal productivity of resources in consumer goods.

Simulation of the two-sector model

With G_t^a as a constant parameter, we have a standard AK^v , $v \in [0,1]$ style growth model. We do not solve for a stationary equilibrium although one is implicit in the structure of the model thus far. Instead, resources are allocated to the two sectors (as if by perfectly competitive agents) in a recursive fashion taking only the current period's marginal product of resources in each line of activity into account to maximize the value of consumer goods production:⁷

$$\begin{aligned} \max_{r_{c,t}, r_{a,t}} c_t &= \mu A_t r_{c,t}^\alpha \\ \text{s.t.} & \\ A_t &= a_t + (1 - \varepsilon) A_{t-1} \end{aligned} \quad [4]$$

Agents in the consumer goods sector pay the opportunity cost of A_t in terms of foregone consumption associated with the resources that go into the R&D sector. The marginal products of resources in the consumer goods and applied R&D sectors are constantly in transition and are determined by the amount of resources devoted to the two sectors and by A_t and G_t^a . However, as A_t is increased over time by the activities of the applied R&D sector, the marginal productivity of resources in the direct production of c rises steadily, causing fewer and fewer resources to be allocated to producing c indirectly through applied R&D. Eventually, A_t becomes so large that the resources devoted to R&D are just enough to compensate for the depreciation of technological knowledge. At this point, there is no further growth.⁸

We use Microsoft Excel to perform the numerical simulations.⁹ The parameterization for the two sector model is as follows:¹⁰

$$\begin{array}{llll} \alpha = 0.3 & \beta = 0.3 & v = 1 & \mu = 1 \\ A_0 = 1 & \varepsilon = 0.02 & R = r_{c,t} + r_{a,t} = 1000 & \end{array}$$

There is nothing special about the parameter values chosen because choosing other values that satisfy our basic assumptions generate qualitatively identical outcomes.

G^a is 1 initially and then changes exogenously to 10 in period 51. Figure 1a¹¹ shows the time series for the value of the output from the consumer goods sector and the accumulated knowledge from the applied R&D. In the fifty-first period the parameter G^a increases. This rejuvenates the growth process because there is an increase in the allocation of resources to the R&D sector (whereas up to then the resources in the R&D sector had been falling).

Figure 1b plots the time series for the allocations of resources in the goods and the R&D sector. As productivity grows in the consumption sector, resources allocated to the R&D sector fall while those allocated to consumption rise. When the exogenous increase in G_t^a occurs in period 51, the marginal productivity of resources devoted to R&D increases, causing a diversion of resources out of consumption into R&D. After this shift, resources are transferred back into consumption over time as productivity in

direct production of c_t rises. This once again slows the growth process, which eventually comes to a halt.

2.2 A Three-Sector Model

We now add the general purpose R&D sector that endogenizes the G_t^a term. In order to focus on GPT-created technological spillovers as drivers of sustained growth, we abstract from two of the important aspects of GPT-driven growth outlined earlier.

First, we assume that there is only one GPT in existence at any one time. This gives our growth process a more homogeneous appearance than the complex reality in which many GPTs typically exist at various stages in their exploitation — sometimes complementing and sometimes competing with each other.

Second, we assume that each new GPT causes an immediate once-for-all shift in the productivity of applied R&D. This creates unrealistic spikes in the growth rates of output, knowledge acquisition and TFP — spikes that have no counterpart in reality. The logistic curve nature of a GPT's applications that we referred to at the outset, could be modeled in our framework by assuming that each new GPT causes the productivity of applied R&D to rise slowly at first, then ever more rapidly, then finally slowing again as the GPT's possibilities are more or less fully exploited. Such a process would be more realistic in removing the immediate spikes and allowing the productivity effects to build up over time.

Although we intend to model both of these phenomena in future papers, the present simplifications are sufficient to move towards more realistic models of GPT driven sustained growth and to reveal some of the problems in using TFP to track the course of technological change.

We also need consider how to model an important feedback from the applied R&D sector to general purpose R&D. In practice, knowledge produced in the applied sector is potentially useful to both the consumer goods sector and the general purpose sector. Examples include;¹²

- Torricelli's demonstration of the weight of air in the atmosphere was of utmost scientific importance and came out of his attempts to design an improved pump.
- Carnot's creation of the science of thermodynamics was an outgrowth of attempts to explain what determined the efficiency of steam engines.
- Joule's discovery of the law of conservation of energy came from work on alternative sources of power for his father's brewery.
- Pasteur's development of the science of bacteriology came out of his studies of fermentation and putrefaction in the French wine industry.

We could model these as unintended spillovers from applied to pure research by allowing the whole stock of applied research, A_t , to enter into the production functions for both consumption and pure research. Instead, use the parameter $\mu \in (0,1)$, which was first introduced in equation [2], to divide A_t between these two sectors.¹³ We make this assumption to ensure that we do not introduce externalities or increasing returns to knowledge (the only accumulating factor) into the model, either of which would affect our calculations of TFP in section 3.

Sector 3, Pure research: This sector produces pure knowledge, g_t , using a production function that has decreasing returns to the resources it employs, $r_{g,t}$, and a random

component, θ_t , to reflect the fact that researchers cannot know in advance how much knowledge will be generated by a given amount of effort:

$$g_t = (1 - \mu)A_t\theta_t r_{g,t}^\sigma \quad [5]$$

where $\sigma \in (0,1)$. Note that if the knowledge of the applied R&D in [5] were treated as an input, the general purpose production function would exhibit constant returns to knowledge and increasing returns to scale, just as in the production functions [2] and [3].

The stock of potentially useful pure knowledge, G_t^p , accumulates according to:

$$G_t^p = g_t + (1 - \delta)G_{t-1}^p \quad [6]$$

where δ is the depreciation rate that reduces the usefulness of pure knowledge accumulated in the past.¹⁴

Although they are potentially useful, new stocks of pure knowledge only become practically useful to the applied R&D sector when a breakthrough occurs in the form of the invention of a new GPT. This happens in our model when the random variable, λ_t , exceeds some threshold value, as shown in equation [7] below. When it does occur, the extra productive power that the new GPT gives to the applied R&D sector depends on how much pure knowledge has been accumulated since the last GPT and a random variable, ϑ_t , which reflects the fact that the applied potential of GPTs varies in ways that cannot be predicted when they are originally being developed. Thus the stock of pure knowledge that is actually useful for applied R&D at any point in time is:

$$G_t^a = \begin{cases} G_{t-1}^a + \vartheta_t(G_t^p - G_{t-1}^a) & \text{if } \lambda \geq \lambda^* \\ G_{t-1}^a & \text{otherwise} \end{cases} \quad [7]$$

where λ^* is the threshold value of λ_t and ϑ_t is a random number that takes on only positive values (many of which can be fractions). ϑ_t determines the affect on R&D of the new GPT when it occurs.

λ_t and ϑ_t are random numbers derived from a beta distribution with support [0,1]. The probability density function for the beta distribution is:

$$\text{beta}(x | \nu, \eta) = \frac{x^{(\nu-1)}x^{(\eta-1)}}{\text{Beta}(\nu, \eta)} \quad [8]$$

where $\text{Beta}(\nu, \eta)$ is the Beta function and ν and η are parameters which take on positive integer values. We choose θ_t from a uniform distribution with a mean of 1 so that on average the resources devoted to the general purpose sector will be as effective as resources elsewhere in the system for all of the simulation runs reported below. In specific realizations, θ_t will cause resources in the pure knowledge sector to be more or less productive than this average.

We choose $\nu < \eta$ so that the concentration of mass in the pdf for λ_t is skewed toward zero, reducing the likelihood of the arrival of a new GPT in any one period.

We also choose $\nu < \eta$ to derive the value of ϑ_t . We choose $\vartheta_t = sx_t$ where s is a calibration parameter in this version of the three sector model and x_t is the random

variable drawn from the beta distribution. $s > 1$ allows ϑ_t to take on values greater than one but only occasionally.

It remains to show what determines the amount of resources, $r_{g,t}$, that will be devoted to pure research. This three sector model includes the pure research sector, which is characterized by uncertainty with respect to when and how much return is associated with resources devoted to it. There is uncertainty because, as we noted in section 1, agents are assumed to be unaware of the distributions that generate the arrival dates and the variation in the size of each GPT. We maintain the assumption implicit in the previous model that utility is monotonically increasing in consumption.¹⁵ But we must now make a further assumption about how agents form expectations of the future consumption payoff from resources devoted to general purpose R&D. For simplicity, we assume that agents only observe the current marginal product of resources going into pure research. The model is then solved recursively with each agent taking as given the marginal products of resources devoted to each sector (or the marginal consumption value of output from the general purpose sector). The period by period maximization in this case is as follows:¹⁶

$$\begin{aligned}
 & \max_{\{r_{c,t}, r_{a,t}, r_{g,t}\}} c_t = \mu A_t (r_{c,t})^\alpha \\
 & s.t. \\
 & A_t = a_t + (1 - \varepsilon) A_{t-1} \\
 & a_t = \nu G_t^a r_{a,t}^\beta \\
 & \tilde{G}_t^p = \tilde{g}_t + (1 - \delta) G_{t-1}^p \\
 & \tilde{g}_t = (1 - \mu) A_{t-1} r_{g,t}^\sigma
 \end{aligned} \tag{9}$$

where \tilde{g}_t differs from g_t because it does not include θ_t (i.e. agents do not recognize that the marginal product of resources devoted to the general purpose sector is subject to random variation), and the actual accumulation of pure knowledge occurs according to:

$$\begin{aligned}
 g_t &= (1 - \mu) A_t \theta_t r_{g,t}^\sigma \\
 G_t^p &= g_t + (1 - \delta) G_{t-1}^p \\
 G_t^a &= \begin{cases} G_{t-1}^a + \vartheta_t (G_t^p - G_{t-1}^a) & \text{if } \lambda \geq \lambda^* \\ G_{t-1}^a & \text{otherwise} \end{cases}
 \end{aligned}$$

Again, we employ Solver iteratively so that allocating resources to each of the three sectors maximizes the objective function in [9] in each period. The production resulting from the resource allocations in the applied R&D and GPT sectors are then fed into the appropriate terms (A_t and G_t) and the maximization exercise with respect to resources is repeated in the next period, and so on.

Simulation in a three sector model

With G_t^a endogenously determined, we now simulate the growth process for the three sectors of the economy. The parameterization for this baseline simulation is as follows:¹⁷

$$\begin{array}{lll}
\alpha = 0.3 & \beta = 0.3 & \sigma = 0.3 \\
A_0 = 1 & G_0 = 1 & R = r_{c,t} + r_{a,t} + r_{g,t} = 1000 \\
\varepsilon = 0.02 & \delta = 0.0001 & \nu = 0.1 \\
\mu = 0.5 & &
\end{array}$$

We chose θ_t from a uniform distribution on the interval $[\cdot 8, 1.2]$ and ϑ_t and λ_t from skewed beta distributions with $\nu < \eta$, ($\nu = 5$ and $\eta = 20$ in both cases). Part a of figure 2 shows the time series of the output from the consumer goods sectors and the stocks of applied and pure knowledge. Part b shows the growth rate of the consumer goods sector and part c shows the resource allocation among the sectors for each of the 100 periods that the model was allowed to run.

When a new GPT arrives, there is an increase in the marginal productivity of resources devoted to the applied R&D sector and thus resources are attracted out of goods production into R&D production.¹⁸ The simulation results are similar to those of the two sector model, except that GPTs are now determined endogenously. The arrival of a new GPT makes applied R&D more productive. R&D rises, increasing the growth in productivity in the consumption good and so causing the growth rate of consumer goods and of the stock of knowledge to increase. The growth rate then falls steadily until another GPT arrives and rejuvenates the growth process.

The most straight forward way to show that our model's growth is sustained in the long run is to eliminate the uncertainty. To do this, let λ_t , θ_t and ϑ_t all be unity in every period. This is equivalent to having the resources that are devoted to pure research yield a return that is solely determined by the production function, having GPTs arrive every period with certainty, and have a uniform impact (i.e., a three sector endogenous growth model). When we run the simulation, the system converges to a constant growth rate where consumption, the stock of applied R&D knowledge are growing at the same rate and the stock of pure knowledge is growing at a slightly lower but positive rate. These results are shown in Figure 3.

2.3 Sustainable growth with decreasing returns to knowledge

The previous version of the model included production functions where new knowledge impacted with a non-decreasing effect on the productivity of the activity that it affected. In this version, we assume that the impact of new knowledge diminishes as its amount increases. We do this by introducing a fractional exponent on the quantity of knowledge where it enters the production function for consumption. Thus, if we were to treat knowledge as an input to production, the production functions would exhibit decreasing returns to ceteris paribus increases in applied knowledge. However, we add the assumption that the current stock of GPT knowledge spills over to increase the potential impact of future GPTs. This assumption is consistent with evidence that increasing knowledge both increases current productivity and creates opportunities for further research. To do this, we endogenize ϑ_t by endogenizing the scaling parameter s_t :

$$\vartheta_t = x_t [s_t] \quad [10]$$

where $s_t = (G_t^a)^\omega$ and $\omega \in [0, 1]$.

We alter the production functions as follows:¹⁹

$$c_t = (\mu A_t)^{\alpha_1} r_{c,t}^{\alpha_2}, \quad \alpha_1 + \alpha_2 = 1 \quad [2']$$

$$a_t = \nu (B_t^a)^{\beta_1} r_{a,t}^{\beta_2}, \quad [3']$$

$$g_t = ((1-\mu)A_t)^{\sigma_1} \theta_t r_{b,t}^{\sigma_2}, \quad [5']$$

Once again, we run the numerical simulation, using Solver to allocate resources among the sectors to maximize equation [9]. The parameter values are as follows:

$$\begin{array}{lll} \alpha_1 = 0.7 & \alpha_2 = 0.3 & \beta_1 = 1 \\ \beta_2 = 0.1 & \sigma_1 = 1 & \sigma_2 = 0.1 \\ \nu = 0.1 & A_0 = 1 & G_0 = 1 \\ \varepsilon = 0.02 & \delta = 0.0001 & R = r_{c,t} + r_{a,t} + r_{g,t} = 1000 \end{array}$$

θ_t and λ_t are chosen as in the previous simulation. ϑ_t is chosen from the skewed beta distribution as above and then multiplied by $(G_t^a)^\omega$ where $\omega = 0.1$. We note that the parameters α_i , β_i , and σ_i for $(i = (1,2))$ have decreasing returns to knowledge only in the consumption sector. This is because in the recursive representation of the objective function each of the knowledge stocks from all lines of activity are subject to the diminishing returns of α_i .²⁰

Part a of Figure 4 shows the plot of the consumption and the knowledge stocks, part b shows the growth rate of consumption and part c shows the resource allocation. The spikes in the plot of the growth rate correspond to the introduction of new GPTs.²¹ There is no pattern in the size of impact that one GPT has relative to another. Large impacts may occur after small impacts, and vice versa, according to the realization of ϑ_t . The important feature of the growth pattern is that GPTs rejuvenate the growth process and there is no trend towards ever-smaller impacts (or the ever larger ones as is often assumed in the literature on “productivity paradoxes”). If the spillover from current to future GPTs was not present, the growth rejuvenation resulting from each subsequent GPT would be smaller and smaller and the growth process would not be sustainable in the very long run. As it is, however, growth is sustainable. This can be verified by removing the uncertainty by setting θ_t , λ_t and ϑ_t equal to one in each period. The system then exhibits unbounded growth as the spillover from the pure research sector implies that there are increasing returns to scale in the pure knowledge sector.²²

With these revised assumptions, we can capture more of the stylized facts presented in Carlaw and Lipsey (forthcoming 2003) where it is argued that GPTs expand the opportunity set for invention and innovation, sometimes even enabling other GPTs. We do this here by employing a model in which the accumulation of applied and pure knowledge yields diminishing returns to output (i.e., no sustained growth by itself) but with a spillover effect that increases the potential of further GPTs. These spillovers increase the potential payoff from resources devoted to new research by increasing the maximum potential size of any future discovery but do not guarantee that further discoveries will be made, or that new discoveries will necessarily be larger than past ones.

3. MEASURING TECHNOLOGICAL CHANGE

In this section we take a step towards providing a theory of TFP by calculating TFP numbers from our simulated data and seeing under what conditions, if any, TFP actually measures technological change.²³ This analysis also illustrates, within the context of our model, some of the more general arguments found in our other two papers in this series. Lipsey and Carlaw (2003, forthcoming) argue that as it is usually calculated, TFP does not measure the effects of technological change itself but only a small subset of the spillovers associated with such change.²⁴ Carlaw and Lipsey (2003, forthcoming) present a detailed analysis of the spillovers associated with technological change and of the subset that the conventional measures of TFP are able to track, at least under ideal conditions.

We calculate and assess a total factor productivity measure from the data generated by our simulations. We always have three outputs, consumption, c , applied research, a , and pure research, g , and the three inputs of resources into these three sectors. Whenever we treat knowledge as an input, we have three additional inputs, one for each sector. Allowing for all of these, our aggregate accounting identity is²⁵

$$p_c c + p_a a + p_g b \equiv q_{rc} r_c + q_{ra} r_a + q_{rg} r_g + q_{Ac} \mu A + q_{Ga} G^a + q_{Ag} (1 - \mu) A \quad [13]$$

where p_i 's, $i \in \{c, a, g\}$ are output prices and q_j 's, with the subscripts $j \in \{rc, ra, rg, Ac, Ga, Ag\}$, are input prices. The first letter of the input price subscripts indicates the input and the second letter the sector in which the input is used. For example, q_{rc} means the price of the resource input used in the consumer goods sector, while q_{Ga} means the price of pure knowledge in the applied R&D sector. We can choose any price as a numeraire and for convenience we choose to measure everything in consumption units. This requires dividing through the identity by p_c to establish relative prices.

$$c + \frac{p_a}{p_c} a + \frac{p_g}{p_c} g \equiv \frac{q_{rc}}{p_c} r_c + \frac{q_{ra}}{p_c} r_a + \frac{q_{rg}}{p_c} r_g + \frac{q_{Ac}}{p_c} \mu A + \frac{q_{Ga}}{p_c} G^a + \frac{q_{Ag}}{p_c} (1 - \mu) A \quad [13']$$

Given the assumption of perfectly competitive equilibrium in each time period (but not a stationary equilibrium over time), the price of resources must be the same in all uses. Letting this common price be q , we can write:

$$\begin{aligned} q &= q_{rc} = p_c MP_{rc} \\ q &= q_{ra} = p_a MP_{ra} \\ q &= q_{rg} = p_g MP_{rg} \end{aligned} \quad [14]$$

which implies :

$$\begin{aligned} \frac{p_a}{p_c} &= \frac{MP_{rc}}{MP_{ra}} \\ \frac{p_g}{p_c} &= \frac{MP_{rc}}{MP_{rg}} \end{aligned}$$

Similarly we can derive input prices relative to the price of the consumption good as follows:

$$\begin{aligned}
\frac{q_{rc}}{p_c} &= \frac{q_{ra}}{p_c} = \frac{q_{rg}}{p_c} = \frac{p_c MP_{rc}}{p_c} = MP_{rc} \\
\frac{q_{Ac}}{p_c} &= \frac{q_{Ag}}{p_c} = \frac{p_c MP_A}{p_c} = MP_A \\
\frac{q_{Ga}}{p_c} &= \frac{p_a MP_G}{p_c} = \frac{MP_{rc}}{MP_{ra}} MP_G
\end{aligned} \tag{16}$$

Resources can be used in all three activities. So the first line of equation [16] shows all of the resource input prices equal to each other and determined by the marginal product of resources in the consumption sector. The input prices of the knowledge stocks are not the same in all production functions because A and G are not substitutes. So, while A used in the consumption sector can be substituted for A used in the pure knowledge sector, G , used in the applied knowledge sector is not a substitute for A anywhere else. The second line of equation [16] shows the price of the applied knowledge stock being equal in both the consumption and pure knowledge sector but a specific adjustment for the price of the stock of pure knowledge must be made in the last line of equation [16] to make it consistent with all other prices.

In the equations that follow, we represent TFP changes as a Divisia index because our theoretical model is specified in continuous time. However, since the data generated by our model are discrete, we use a Tornquist index to calculate TFP. Our model allows us to measure the rate of growth of technology directly as the rate of change of the knowledge stocks, A_t , and, G_t^a .

Letting Y_i 's represent the outputs of the three sectors and X_j 's represent their inputs, the Divisia index of TFP changes is:

$$\frac{\dot{TFP}}{TFP} = \frac{\dot{Y}}{Y} - \frac{\dot{X}}{X} = \sum_i w_i \frac{\dot{Y}_i}{Y_i} - \sum_j v_j \frac{\dot{X}_j}{X_j} \tag{17}$$

where

$$w_i = \frac{p_i Y_i}{\sum_i p_i Y_i} \quad \text{and} \quad v_j = \frac{q_j X_j}{\sum_j q_j X_j} \tag{18}$$

TFP can now be calculated using various assumptions about how outputs from the pure knowledge sector are observed, about how knowledge is treated in the production function, and about scale effects.

There are three ways in which the output of the pure research sector could be measured: the actual flow of output from the pure research sector, g_t , the input costs of creating that knowledge, $r_{g,t}$ and $(1 - \mu)A_t$ and as zero.²⁶ There are two ways of treating the influence of knowledge: as an input into the relevant production functions and as a shift parameter in those functions. When knowledge is treated as an input, the assumption about returns to scale matters. When knowledge is treated as a shift parameter, the scale assumption is irrelevant. The combination of these possibilities gives rise to nine of the cases shown in Table 1: the three cost assumptions combined with the two scale assumptions, when knowledge is an input, give six cases while the three cost assumptions combined with treating knowledge as a shift parameter yield the other three.

In addition to the nine cases identified so far, there are three other cases that we consider. The first two cases have no uncertainty but differ on the assumption of returns to scale. In one case there are constant returns to knowledge (IRS) and in the other there are decreasing returns to knowledge in the consumption sector. The output of pure research is measured directly and knowledge is included as an input in the TFP calculations in both. The third case is just like the first one except in this case knowledge is not included as an input in the TFP calculations. We report only the first and third cases as the second is qualitatively the same as the first. This is true even though the returns to knowledge assumptions differ because the case where knowledge has decreasing returns in the consumption sector has a spillover operating where the accumulation of past pure knowledge increases the all production of future pure knowledge.

There are two reasons for reporting the no uncertainty cases. The first is to provide a formal rationale for the verbal arguments of Lipsey and Carlaw (2003) and Carlaw and Lipsey (2003). The first paper argues that TFP does not necessarily track technological change because costly technological change carries no necessary free lunch with it. The second paper argues that technological advance need have no contemporaneous externalities associated with it and therefore will not necessarily generate positive TFP change. Both of these arguments are bourn out in the first case of no uncertainty considered here.

The second reason for reporting the two no uncertainty cases is related to the constant returns to knowledge cases where the output from the pure knowledge sector was measured at either its true flow or at its resource costs. The no uncertainty case with constant returns to knowledge allows for a more complete analysis of the uncertainty cases with constant returns to knowledge. We say more about this below table 1.

There are thus twelve possible cases, nine of which are reported in the panels of figure 5. All of these possibilities are summarized in Table 1

Table 1.

Measurement of the output of pure knowledge	Constant returns to knowledge as an input	Decreasing returns ²⁷ to knowledge as an input
	(1st 3-sector model)	(2nd 3-sector model)
Measured directly	Case 1, <i>panel a</i>	Case 2, <i>panel b</i>
Measured at cost	Case 3, <i>panel c</i>	Case 4, <i>panel d</i>
Measured directly & no uncertainty ²⁸	Case 5, <i>panel e</i>	Case 6, not reported
Not measured	Case 7, not reported	Case 8, <i>panel f</i>
<u>Knowledge is not treated as an input²⁹</u>		
Measured directly		Case 9, <i>panel g</i>
Measured at cost		Case 10, not reported
Not measured		Case 11, <i>panel h</i>
Measured directly & no uncertainty		Case 12, <i>panel i</i>

In *panel a* of Figure 5, the output flow from the pure research sector is measured directly and there are constant returns to knowledge in all of the production functions. After an initial spike down (due to the way the GPT enters the system), TFP growth spikes back up and thereafter is positively correlated but is less than the growth rate of the pure and applied knowledge stock.³⁰ TFP growth tracks knowledge growth because the increasing returns to scale (i.e., across both resources and knowledge) in the production functions of each sector imply that output is growing faster than inputs.

Panel b of Figure 5 is the case where pure research output is measured at its output flow and there are constant returns to knowledge. In this case, TFP growth initially spikes down and then back up (again as a result of the way in which the GPT enters the system). However, after the GPT has arrived, the TFP growth rate is negative and negatively correlated with the actual growth of knowledge. This case differs from the previous one because of the different assumption about returns to scale in the production function. The change from constant (*panel a*) decreasing (*panel b*) returns to knowledge causes the TFP number to switch from tracking the actual growth of technological knowledge in the system (after the GPT arrives) to being negative and negatively correlated with knowledge growth.

Panel c of Figure 5 presents the case in which the output from pure research is measured at its input cost and there are constant returns to knowledge in all production functions. TFP growth in this case again spikes down when the GPT first arrives. It then spikes up, and is thereafter positive and positively correlated with technological change being driven by the accumulation of knowledge from the applied R&D sector. The result is similar to that in *panel a* because the returns to scale assumption in the production functions is the same in both cases.

Panel d of Figure 5 shows the case in which the output from the pure knowledge sector is measured at inputs cost and there are decreasing returns to knowledge in the consumption production function. The result is similar to that of case 2, *panel b*, where after an initial spike down, then up, the TFP growth rate becomes negative and negatively correlated with knowledge growth.

Panel e of Figure 5 illustrates two important points. First, it helps to clarify the measurement of TFP in the first four cases. In this case (5) the returns to knowledge are constant and output from the pure knowledge sector is measured at its actual flow. The difference between this and case 1, *panel a*, is that there is now no uncertainty in the system. TFP is zero while knowledge growth is positive. Notice that in the previous two cases where knowledge has constant returns in the production functions TFP at least tracks the knowledge growth rate in the periods where no GPTs arrived. In this case, TFP does not track knowledge accumulation. In the cases where there are decreasing returns to knowledge in the production function for consumption, TFP was negative and did not track knowledge growth. In case 6 (that we do not report a figure for) where there are decreasing returns to knowledge in the consumption sector TFP converges to negative constant rate of change while knowledge growth is everywhere positive.

The second important point illustrated by case 5 is that it formally demonstrates the verbal arguments presented in Lipsey and Carlaw (2003) and Carlaw and Lipsey (2003) that costly technological change in a system that exhibits no contemporaneous externalities or increasing returns to the accumulating factors can result in zero TFP growth. It is for this reason that we include the growth rate of consumption in *panel e* to show that sustained growth driven by technological change is taking place while TFP growth is zero. Thus, technological change and rising real income can occur in the absence of positive TFP growth. To interpret zero TFP growth as evidence of no technological change is thus very misleading. Furthermore, in cases 2, 4 and 6³¹, positive technological change is occurring while TFP change is actually negative.

Panel f shows the case in which there is no output measure associated with the pure research sector. Although the case is shown for the model with the diminishing returns to knowledge, the qualitative result holds even for the case of constant returns to knowledge (IRS) production functions. Omitting any measure of output of the pure research sector results in a TFP pattern similar to that of *panels b* and *d*. When the GPT arrives, TFP spikes down and becomes negative and then spikes up to become positive before settling down to be thereafter negative and negatively correlated with knowledge growth. This makes intuitive sense since all that has been done in comparison to the first four cases is to decrease the output measure holding all input measures constant.

Panels g and *h* show the results when knowledge is not counted on the input side and uncertainty prevails in the system. *Panel g* includes the actual value of the output flow from the pure research sector and *panel h* has no measure of output from this sector. (Once again we do not provide a figure for case 10 because it is qualitatively the same as case 9.) In *panel g* we see that when the GPT is introduced, TFP overshoots actual growth but thereafter TFP growth is coincident with knowledge growth. In *h* we see again that when the GPT is introduced, TFP overshoots actual knowledge growth and then TFP growth is positively correlated with, but everywhere less than, knowledge growth. The obvious explanation for the difference between *g* and *h* is that we have

not measured all of the output contributions thus TFP under reports technological change.

Panel i shows the case where knowledge has constant returns in all production functions, knowledge is not measured as an input and there is no uncertainty in the system. In this case, TFP and knowledge growth converge to be equal and positive constants. Only in this case does TFP growth track knowledge growth exactly.

What do these simulations tell us about what TFP actually measures? In *panels a* and *c*, we see that TFP growth measures something positive, but does it measure technological change as measured by our growth of pure and applied knowledge? Because of the constant returns to knowledge, increasing returns in production and the correctly measured output from the pure research sector, TFP growth correlates with actual technological growth. However, the TFP calculations where the flow of output from the pure knowledge sector is measured directly are operationally impossible to implement since the output from the pure research sector cannot be directly observed when there is uncertainty. The only thing observable on the output side is the impact of a GPT after it arrives. Furthermore, in the case where the flow of output from the pure knowledge sector can be observed because there is no uncertainty (case 5), TFP does not measure the change in technology shown as the growth of pure and applied knowledge.

A feasible alternative to the set of TFP measures that include the output flow from the pure knowledge sector is to approximate the contemporaneous value of output by its input costs, as we did in cases 2 and 4 (*panels b* and *d*). However, in these cases, TFP does not correctly measure technological change as measured by the growth in pure and applied knowledge. The patterns of technological knowledge and TFP growth in *b* and *d* are similar because the assumption about the underlying production functions is the same.

When we measure TFP as in case 8, *panel f*, as many statistical agencies do, TFP does not measure technological growth. Although we do not report a figure for case 7 this result holds both for diminishing and constant returns to knowledge.

Finally, in all three cases when we do not include knowledge as an input in production, TFP comes closer to measuring technological change (measured as growth in pure and applied knowledge). In the case when we include the contemporaneous output from the pure research sector (*panel g*), TFP growth overshoots knowledge growth when the GPT is introduced but then tracks it exactly. TFP growth again overshoots when the GPT is introduced but then is positively correlated and under estimates actual knowledge growth where we do not include the contemporaneous measure of output from the pure research sector (*panel i*). TFP is perfectly correlated with knowledge growth when there is no uncertainty and knowledge is not treated as an input in the TFP calculation.

There are five measures of TFP growth that correlate positively with actual pure and applied knowledge growth. The first two measures, presented in *panels a* and *c*, are unrealistic because they use a direct measure of the value of potentially useful pure research that is not actually observable in any real world economic data. The other three measures are those presented in *panels g*, *h* and *i*. In these cases, knowledge is not counted as an input to production, even though its own production has an opportunity cost in the sense that it uses existing knowledge and resources that could have been used to produce consumer goods. In these cases, knowledge could be

interpreted to be entirely disembodied. However, once again our ability to measure knowledge directly is an artifact of the simplified experiment conducted here. Real world data very likely has knowledge capitalized in the value of human and physical capital at least to some degree because it occurs with a resource cost.

4. CONCLUSION

In this paper we have built a model of GPT driven growth that is simultaneously technically simpler than the existing models while containing several assumptions and some types of behavior that are closer to what is observed empirically about GPTs than is found in existing models. We have also shown that calculated rates of TFP growth do not necessarily measure technological progress, especially in those cases where the value of knowledge gets capitalized into the value of inputs.

4.1 Conforming with the stylized facts

The main assumptions and behavior patterns of our model that come closer than the first generation of GPT model to observations about actual GPTs are summarized as follows. Separate sectors endogenously generate fundamental research that help to create new GPTs and knowledge that applies the existing GPTs to improve methods of producing consumption goods. The future history (path of development) of any given GPT cannot be known as soon as it arrives so agents make maximizing decisions with respect to current information about the marginal products of resources used in each sector. They are unaware of the spillovers that result from their current allocation decisions on future changes in these marginal productivities. There is growth fluctuates because GPTs have variable life times and variable effects in increasing income. Thus the rate of income growth is not constant from one GPT to another; neither does it always accelerate or decelerate over time.

4.2 Growth and TFP Change

We meet our second objective by using our three sector model to generate data from which TFP calculations can be made on a variety of assumptions about returns to scale and about what is counted as inputs and as outputs. Note that in the cases presented in *panels g, h* and *i* of Figure 5, where TFP actually measures technological growth, knowledge is separable from other inputs and the value of knowledge has not been capitalized into the value of other inputs — which Lipsey and Carlaw (2003, forthcoming) argue often happens in practice. These calculations were feasible because we had generated data that allowed us to measure knowledge accumulation independently from other economic activity. Our assumption that knowledge is the only "factor" being accumulated allows us to distinguish explicitly between cases where knowledge is capitalized into the value of inputs (i.e., “embodied” in Chen’s (1997) sense) and cases where knowledge is not so capitalized (i.e., “disembodied” in Chen’s (1997) sense). It remains an open question that is the subject of our further research as to which of our measures comes closest to those in practical use when TFP is calculated from real data where several different categories of human and physical capital are being accumulated along with knowledge, and these forms of capital embody much of the knowledge being accumulated in the economy.

We should also point out that the problems with TFP that we have studied in this paper are in addition to many of the problems studied by Lipsey and Carlaw (2002, forthcoming) because we are dealing here with a theoretical model in which we know

exactly how the data were generated while they dealt with many of the practical problems associated with the data actually used.³²

4.3 Further Modeling Possibilities

The following are improvements that can be made in order to get closer to the empirically observed behavior of GPTs. Fortunately, compared with most other types of endogenous growth models, the recursive nature of our model, in which an balanced equilibrium growth path is never reached, makes it easier to allow for behavior that is more realistic but also more complex. First, the instantaneous effect of GPTs in raising the productivity of applied research needs to be removed. To do this a lag can be introduced so that a new GPT shifts the marginal product curve of applied R&D according to a logistic time series, slowly at first, then more and more rapidly, then slower. Second, more than one GPT can be allowed to exist simultaneously. It will then be possible to have some GPTs arrive when the possibilities of the GPT it replaces are fully exploited and others when unexploited possibilities remain. Both cases have been observed in history. Helpman and Trajtenberg's models have different results in each of these cases. But because they were looking for a steady state growth path, they had to choose the constant period of the GPTs life to allow for only one of these two possibilities. Third, a new GPT could be assumed to require some minimum accumulation of new pure knowledge since the previous GPT was introduced. In this case, the conditions for the arrival of a new GPT would be two fold: first, a given amount of pure knowledge must be accumulated, second, as in the present model, a random drawing of values for λ must exceed its critical value. Fourth, GPTs could become easier to invent and/or more powerful as the stock of pure knowledge accumulates (a type of network externality in knowledge accumulation). In this case, either or both the arrival date and the GPT-induced shift in the marginal product of the applied R&D sector would be an increasing function of the stock of pure knowledge. This would be an improvement on A&H(T), which is the only model of those surveyed in which the arrival date of new technology is partly endogenous. In their model, the arrival rate is influenced only by contemporaneous factor allocations, whereas historical evidence suggests that it is a function of accumulated knowledge. Fifth, uncertainty can be introduced with respect to applied R&D as is the case in Aghion and Howitt's GPT model. The applied R&D sector could be made to exhibit randomness by making the marginal product of resources in applied R&D subject to a random component similar to what we now have in our pure research sector. Sixth, in our model an increase in applied knowledge affects the production function in consumption instantaneously. A lag would be more realistic where what is learned about applied knowledge in this period affects the production function of consumption goods in some subsequent period. This would allow for lags, which, together with the logistic effect of a new GPT on the production function of the applied R&D sector, could easily produce slowdowns associated with the introduction of some GPTs and not with others. Since slowdowns are observed to vary among GPTs empirically, the possibility of having the slow down associated with new GPTs in theoretical models vary with observable parameters and the endogenous stock of accumulated endogenous variables is potentially powerful.

END OF TEXT

NOTES

* This is a revised version of the second half of the paper “Externalities versus Technological Complementarities: a model of GPT-driven sustained growth” delivered at the conference in honor of the 20th anniversary of Nelson and Winter’s *Evolutionary Theory of Economic Change*, Aalborg Denmark 12-15 June 2001. We are indebted for comments and suggestions to Alvero Pereira, Dieter Ernst, and Gerald Silverberg, as well as to the members of the Canadian Institute for Advanced Research’s economic Growth and Policy Program, and the participants of the Waikato Growth Workshop sponsored by the Econometric Society Australasian Standing Committee (ESASC) to whom earlier versions of the Aalborg paper were presented. This research was supported in part by a grant from the Royal Society of New Zealand’s Marsden Fund (grant number UOC 101).

¹ Lipsey, Bekar and Carlaw (1998a and 1998b) have studied the economic, social and political effects of some of the most important general purpose technologies (GPTs) from history, grouping them into five main classes. GPTs closely parallel the concept of changes in technoeconomic paradigms as analyzed by Perez (1983) and Freeman and Perez (1988)

² One important implication of this last point is that the common expectation of a productivity bonus following on the introduction of a new GPT (leading to a “New Economy”) has no basis in either fact or theory. See Lipsey (2002).

³ This can be done in either of two ways. First, we can assume that agents are not forward looking, which implies that they only worry about the current period’s marginal products. Second, we can assume that agents are forward looking but do not foresee changes in the marginal products in all lines of production, which implies that they perform the dynamic programming problem each period taking the marginal products in all lines of activities as being constant at the current period level. In the subsequent period they repeat the procedure with the new marginal products encountered that period. Since in our model these amount to the same thing qualitatively, we adopt the first assumption which is simpler.

⁴ Although we have referred to the models in Helpman (1998) as the first generation of GPT models, they actually own some heritage to Besnahan and Trajtenberg (1995). The characteristics of imperfect competition and exogenous arrivals of GPTs in the models in the comparison set are also present in Besnahan and Trajtenberg’s seminal paper on GPTs.

⁵ The empirical evidence about the behavior of GPTs that leads us to take this position is discussed in some detail in Lipsey, Bekar and Carlaw (1998a and 1998b). While some very general qualitative properties can be established for all GPTs, such as that the applications tend to follow a logistic time path, nothing specific enough to allow maximizing over the life time of a new GPT, or even say over its first decade or two, can be inferred from the experience of past GPTs.

⁶ In Lipsey and Carlaw (forthcoming 2003), we study the consequences of the assumption used in virtually all aggregate production functions that resources such as land and raw materials, which cannot actually be accumulated, are included along with physical capital, which can be accumulated, in the composite variable called capital that *is* assumed to be capable of unlimited accumulation. Among other things, we show that technological change that increases the productive efficiency of fixed but unmeasured natural resources will tend to show up as increases in the quantity of measured accumulating factors in an assumed constant returns production function, even though the accumulating factors are subject to decreasing returns.

⁷ We solve this two sector version of the model recursively even though there is an obvious, stationary (steady state) equilibrium over the infinite horizon, because this recursive technique is employed in the three sector versions of the model which follows and in those versions of the model there is no implicit stationary equilibrium.

⁸ Romer, 1996, chapters three and four provides a detailed analysis of this type of model.

⁹ Excel has a built-in routine called Solver that employs the Generalized Reduced Gradient (GRG2) non-linear optimization code developed by Leon Lasdon, University of Texas at Austin, and Allan Waren, Cleveland State University. Portions of the Microsoft Excel Solver program code are copyright 1990, 1991, 1992, and 1995 by Frontline Systems, Inc. Portions are copyright 1989 by Optimal Methods, Inc. We simply iterate through the maximization problem above using Solver to allocate resources in each period.

¹⁰ The parameter values presented in each of the simulations that follow were chosen to yield illustrative cases. Robustness tests have been conducted to see if variations in parameter values alter the qualitative results in each simulation. Where parameter values are important they are discussed in the text. Otherwise the results presented are robust for wide ranges of parameter values. Symmetry in exponential parameters of the production functions is imposed so as to focus attention on the specific qualitative results and make the analysis simpler.

¹¹ All of the figures are presented at the end of the paper.

¹² All of these examples are drawn from Rosenberg (1982, Chapter 7).

¹³ We could also endogenize μ , allowing agents to choose how much of the stock of applied R&D to allocate to the two sectors. However, in versions of the model where we have done this there is no qualitative change to the model and our assumption is simpler.

¹⁴ For now, the process that renders this knowledge obsolete is treated as exogenous. A more sophisticated version of the model would make obsolescence endogenous and a function of the arrival of new GPTs. Howitt (1998) discusses the impact that new GPTs might have on rates of obsolescence in the economy. Carlaw (2000) discusses endogenous (optimal) obsolescence in the context of a microeconomic production function.

¹⁵ So far, utility has been implicit in the maximization of consumption, which requires either that individual agents' utility, or social utility, be monotonically increasing in consumption. Applied and pure knowledge can contribute to increases in consumption but there is an opportunity cost in terms of consumption foregone today for more consumption tomorrow.

¹⁶ This recursive formulation of the objective function is actually a simplification from reality in that the effect of changes in the stocks of knowledge are registered simultaneously in the sector that produces it as an output and in the sector that it affects. Because of the discrete nature of the problem there should actually be a three period lag that takes account of the fact that applied and general purpose knowledge produced in a given period can only be used in the sector in which it enters the production function in the subsequent period. The assumption that there is no lag is also the reason that consumption does not fall in the period where the GPT is realized. If there were a lag, the marginal product of resources devoted to consumption would not immediately reflect the change in knowledge that had shown up in the applied R&D sector and the migration of resources out of consumption into applied R&D would be greater causing consumption to fall. Because of the immediate spread of the effect of new knowledge across all sectors, consumption does not fall because few resources migrate out as the knowledge impacts the system.

¹⁷ Once again, the parameters are chosen to yield illustrative results and all of the conclusions drawn hold qualitatively for a variety of parameterizations.

¹⁸ Depending on the parameterization of the model, resources may also be drawn out of the pure research to the applied R&D sector.

¹⁹ The production function for consumption now exhibits decreasing returns to the knowledge and constant returns to scale if knowledge is treated as an input. The recursive nature of the model implies that all knowledge in the objective function will be subject to diminishing returns.

²⁰ Substituting all of the constraints into the objective function yields:

$$c_t = \left\{ \mu \left[v \left(((1-\mu)A_t)^{\sigma_1} (r_{g,t})^{\sigma_2} + (1-\delta)\tilde{G}_{t-1}^p \right)^{\beta_1} (r_{a,t})^{\beta_2} + (1-\varepsilon)A_{t-1} \right] \right\}^{\alpha_1} (r_{c,t})^{\alpha_2}$$

which has α_1 being applied to all of the knowledge stocks. Thus, if $\alpha_1 < 1$ then knowledge has diminishing returns in the system even though $\beta_1 = \sigma_1 = 1$.

²¹ As noted earlier, the spikes in growth rate of consumers' goods are due to the unrealistic assumption that the GPTs full impact is immediate. (I.e., there is no diffusion process associated with the introduction of a new GPT.)

²² Specific parameterizations of the exponents on knowledge in the production functions matched with specific values of $0 < \omega \leq 1$ can be made such that balanced growth will occur.

²³ In what follows, we use technological knowledge, pure and applied knowledge and technology interchangeably. In our model it is the technological knowledge generated in the pure and applied knowledge sectors that is technological change which drives growth.

²⁴ They are, of course, not the first to argue that TFP does not measure technological change. See, for example, Jorgenson and Griliches (1967) and Hulten (2000). However, Lipsey and Carlaw (2003) go further than these other authors by arguing that TFP is only an imperfect measure of a small subset of the spillovers associated with technological change and that sustained growth with zero TFP change is possible.

²⁵ Some of the TFP calculations we present will not involve all of these items but we define the appropriate input prices for all of the possible inputs in the various treatments of knowledge in the TFP calculations that will be presented below.

²⁶ The reason we consider various ways of measuring output from the pure research sector is because of the uncertainty there. Measurers cannot actually observe the flow of potentially useful knowledge that is accumulated but not released until the arrival of the GPT.

²⁷ Here constant and decreasing returns refer to knowledge in the production function when it is treated as an input.

²⁸ The no uncertainty cases are numbered 5 and 6 and follow on from the first four cases because they provide important information that allows for the analytical interpretation of the first four cases.

²⁹ Only when knowledge is treated as an input, is there a distinction with respect to returns to scale in the two models. In these cases, therefore, the DRS refers to the input of resources in the production functions

³⁰ Between the arrivals of each GPT this rate is being driven only by accumulation of output from applied research as pure knowledge is only useful in production with the arrival of a GPT, even though productivity is being calculated each period.

³¹ Although we did not report this case when TFP was calculated it was negative.

³² In principle, as also argued in the other two papers in this series, technological advance is better measured by the counterfactual comparison of actual income with what it would have been if the technological advance had not occurred. Given the enormous set of technological complementarities that accompany any technological advance, such as the railroads or electricity, this is an almost impossible task in practice (and is not accomplished by the sort of calculations done by cleometricians). For an important explanation of why cleometric measurements do not capture the major effects of major new technologies see Chapter 1 of Freeman and Louca (2001).

REFERENCES

- Aghion, P. and P. Howitt (1998a). "On the Macroeconomic Effects of Major Technological Change", in Helpman (1998), 121-144.
- _____ (1998b). Chapter 2: "The Schumpeterian Approach", *Endogenous Growth Theory*, (Cambridge, Mass.: MIT Press), 53-84.
- Bresnahan, T.F. and M. Trajtenberg (1995) "General purpose technologies 'Engines of growth?'" , *Journal of Econometrics*, 65: 83-108.
- Carlaw, K.I. and R.G. Lipsey, (2003, forthcoming). "Externalities, Technological Complementarities and Sustained Economic Growth" *Research Policy*.
- Carlaw, K. I. (2000) "Optimal Obsolescence Driven by Technological Change" Chapter 3 Ph.D. dissertation *Uncertainty and Complementarity Lead to Scale Effects in the Production of Capital*, Simon Fraser University.
- Chen, E. K. Y. (1997). "The Total Factor Productivity Debate: Determinants of Economic Growth in East Asia," *Asian Pacific Literature*, 11, 18-38.
- Freeman and Louca. (2001). *As Time Goes By: From the Industrial Revolutions to the Information Revolutions*, (Oxford: Oxford University Press).
- Freeman, C. and Perez, C. (1988). Structural crisis of adjustment: business cycles and investment behaviour, Chapter 3 in *Technical Change and Economic Theory*, G. Dosi, C. Freeman, R. Nelson, G. Silverberg and L. Soete (eds.) (London: Pinter).
- Griliches, Z. (1994). "Productivity, R&D, and the Data Constraint", in *American Economic Review*, 84, 1-24.
- Helpman, E. (ed) (1998). *General Purpose Technologies and Economic Growth*, (Cambridge, Mass.: MIT Press), 55-84.

Helpman, E. and M. Trajtenberg (1998a). "A Time to Sow and a Time to Reap" in Helpman (1998), 55-84.

_____ (1998b). "Diffusion of General Purpose Technologies" in Helpman (1998), 85-120.

Howitt, P. (1998) "Measurement, Obsolescence and General Purpose Technologies" in Helpman (1998), 219-52.

Hulten, C. R. (2000) "Total Factor Productivity: A Short Biography", *NBER Working Paper*, **7471**, National Bureau of Economic Research: Washington.

Jorgenson, D. and Z. Griliches (1967). "The Explanation of Productivity Change", *Review of Economic Studies*, 34(3) 249-83.

Lipsey, Richard G. (2002). "The Productivity Paradox: A case of the emperor's new clothes," *ISUMA, The Canadian Journal of Policy Research, Special issue on Policies for the New Economy* 3(1), 120-126 .

Lipsey, R.G. and K.I. Carlaw, (2003, forthcoming). "The Measurement of technological Change", forthcoming in a Statistics Canada monograph *Measuring Productivity and Technological Change*.

Lipsey, R. G., C. Bekar and K. I. Carlaw (1998a). "What Requires Explanation?" in Helpman (1998), 15-54.

_____ (1998b). "The Consequences of Changes in GPTs" in Helpman (1998), 193-218.

Perez, C., (1983). "Structural change and the assimilation of new technologies in the economic and social systems", *Futures*, **15**(5), 357-375

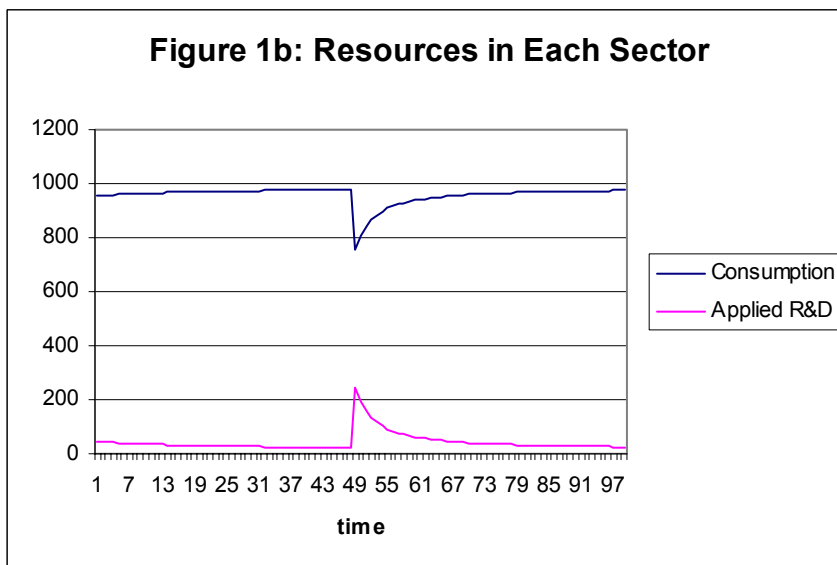
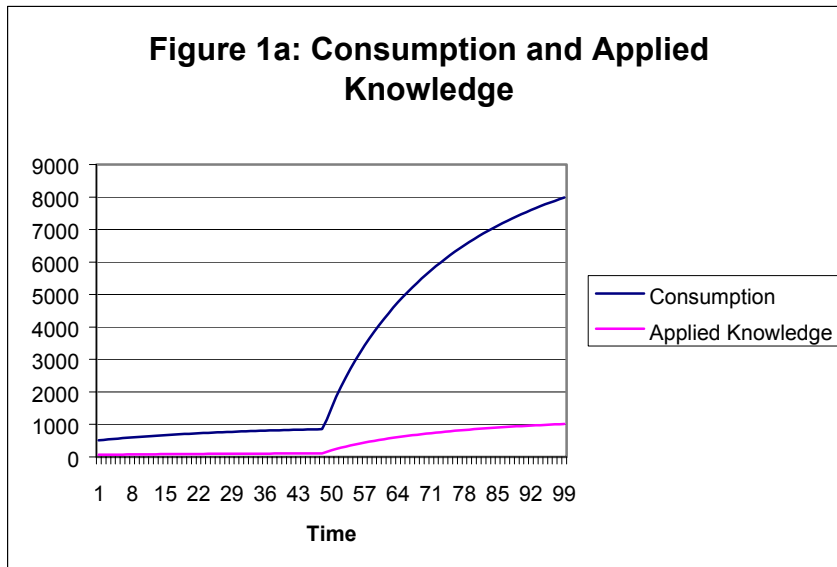
Pfaffenberger, R. C. and J. H. Patterson (1987). *Statistical Methods for Business and Economics*, (Illinois: Irwin).

Prescott, E.C., (1998). "Needed a theory of Total Factor Productivity" *International Economic Review*, **39**, 525-51.

Romer, D. (1996). *Advanced Macroeconomics*, (New York : McGraw-Hill Companies).

Rosenberg, N. (1982). *Inside the Black Box: Technology and Economics*, (Cambridge: Cambridge University Press).

Figures



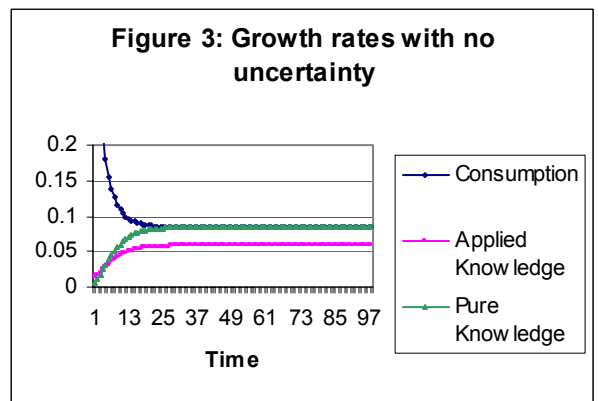
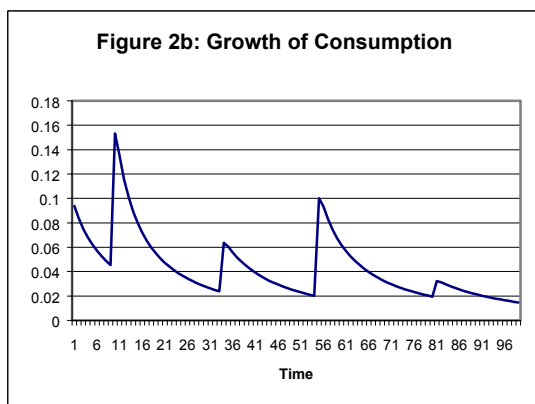
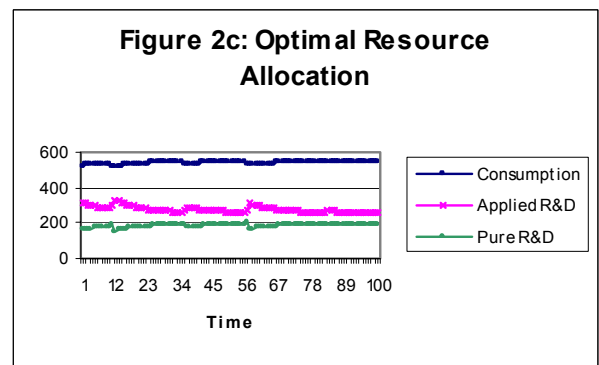
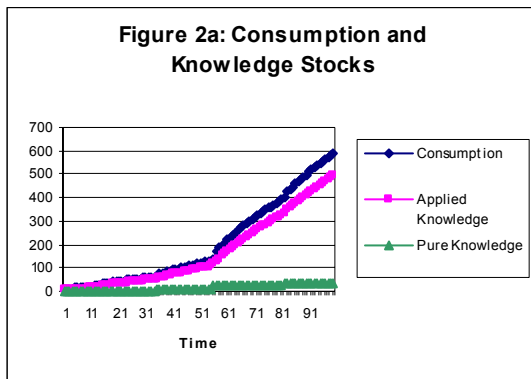


Figure 4a: Consumption and Knowledge Stocks

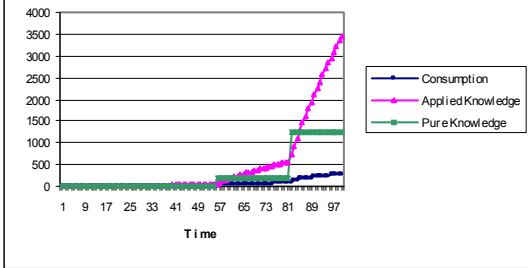


Figure 4c: Optimal Resource Allocation

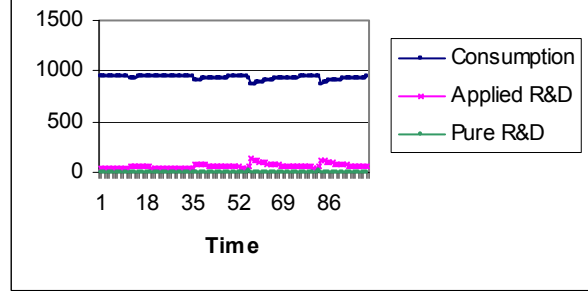
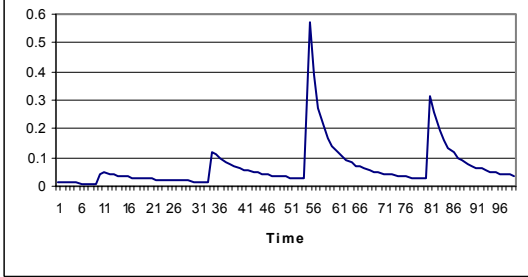
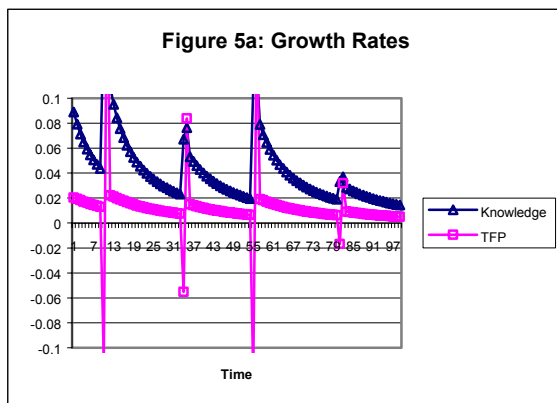
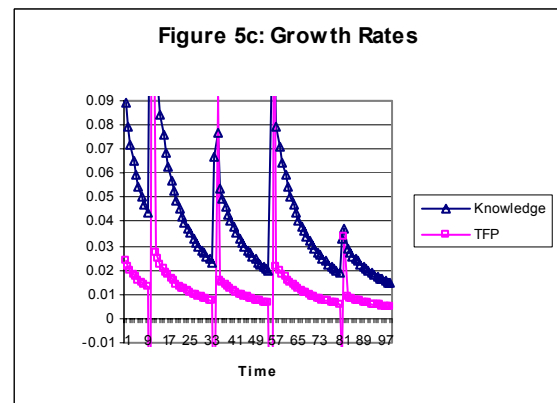


Figure 4b: Growth of Consumption

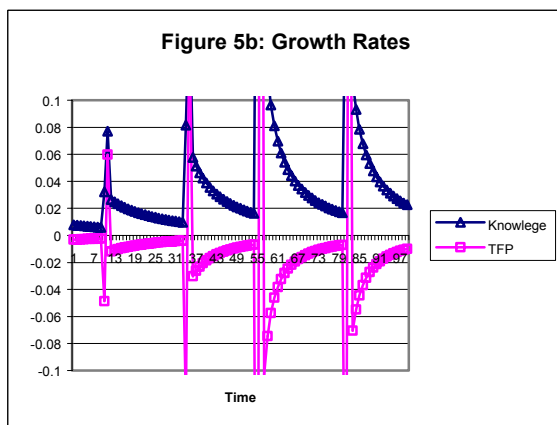




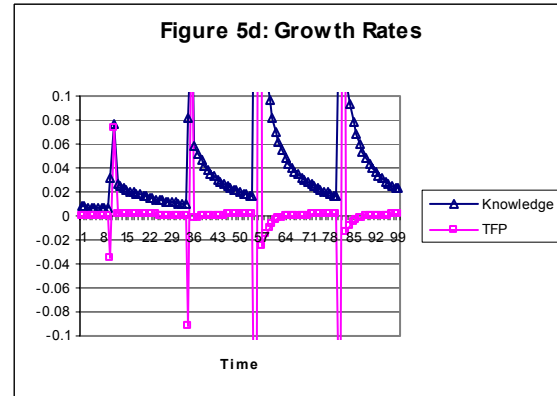
TFP growth rates are calculated with the assumptions of constant returns to knowledge (IRS), knowledge treated as an input and the output from pure R&D sector measured directly.



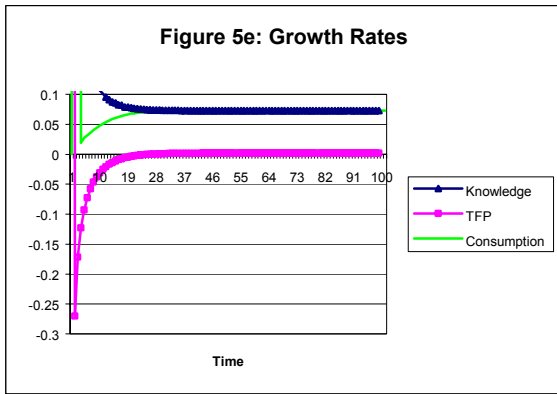
TFP growth rates are calculated with the assumptions of constant returns to knowledge (IRS), knowledge treated as an input and the output from pure R&D sector measured at its input costs.



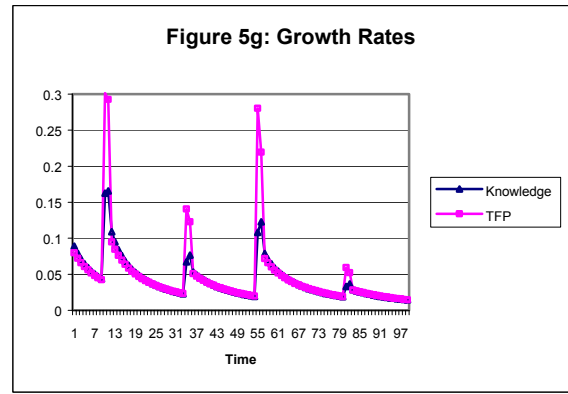
TFP growth rates are calculated with the assumptions of decreasing returns to knowledge in the consumption sector (CRS), knowledge treated as an input and the output from pure R&D sector measured directly.



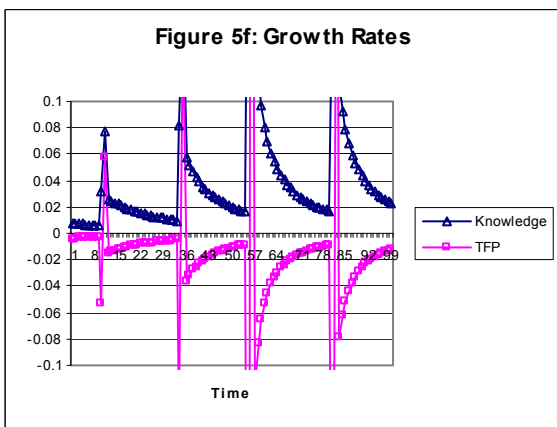
TFP growth rates are calculated with the assumptions of decreasing returns to knowledge in the consumption sector (CRS), knowledge treated as an input and the output from pure R&D sector measured at its inputs costs.



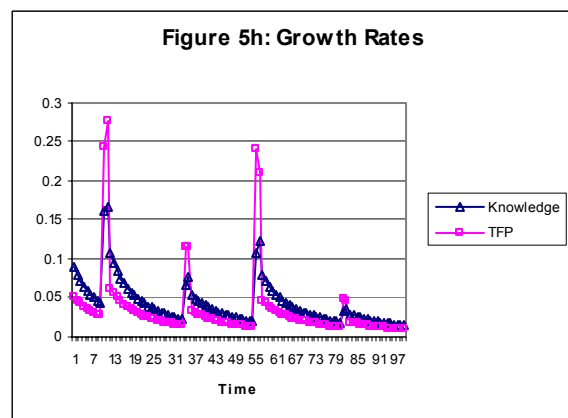
TFP growth rates are calculated with the assumptions of constant returns to knowledge (IRS), knowledge treated as an input, the output from pure R&D sector measured directly and no uncertainty. Consumption growth is included to show that growth from technological change occurs even when TFP is zero.



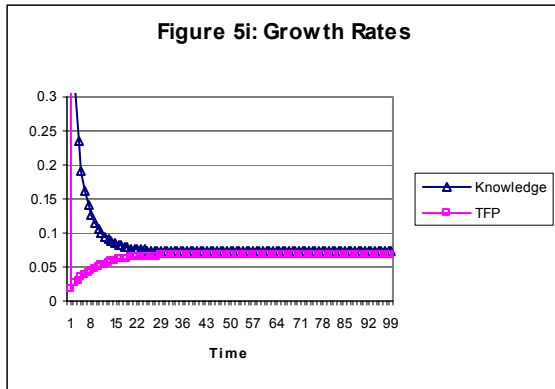
TFP growth rates are calculated with the assumptions of constant returns to knowledge (IRS), knowledge not treated as an input and the output from pure R&D sector measured directly.



TFP growth rates are calculated with the assumptions of decreasing returns to knowledge in the consumption sector (CRS), knowledge treated as an input and the output from pure R&D sector not measured.



TFP growth rates are calculated with the assumptions of constant returns to knowledge (IRS), knowledge not treated as an input and the output from pure R&D sector not measured.



TFP growth rates are calculated with the assumptions of constant returns to knowledge (IRS), knowledge not treated as an input, the output from pure R&D sector measured directly and no uncertainty.