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Abstract. Ownership positions in large corporations can be traded on anonymous markets, but professional partnerships and worker cooperatives prohibit members from transferring their positions to outsiders without the consent of other insiders. These contrasting policies can be explained by adverse selection, which implies that the joint payoff of the partners is at least as large when continuing rather than departing members choose the terms on which new partners can join. In a separating equilibrium, or a pooling equilibrium in which only low-quality workers apply for positions, market sorting reduces total surplus. The market can sometimes improve on random assignment of workers to firms when there is a pooling equilibrium in which both high- and low-quality types apply.

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Partnership Markets with Adverse Selection

> "[I]n a private copartnery, no partner, without the consent of the company, can transfer his share to another person or introduce a new member into the company. Each member, however, may, upon proper warning, withdraw from the copartnery and demand payment from them of his share of the common stock. In a joint stock company, on the contrary, no member can demand payment of his share from the company; but each member can, without their consent, transfer his share to another person, and thereby introduce a new member." (Adam Smith, [1776] 1994: 799)

1. Introduction

In the above passage from The Wealth of Nations, appearing just before his famous diatribe against the joint stock company, Adam Smith distinguishes a partnership from a corporation by highlighting the differing procedures through which members are replaced. This distinction still applies. Someone who owns shares in Microsoft can sell those shares on a stock exchange, transferring voting rights to another party in the process, without first obtaining permission from other shareholders. By contrast, professional partnerships in law or medicine almost never allow departing members to choose their own successors.

In the English common law tradition, systematized by the Partnership Act of 1890, partnerships have no legal identity distinct from that of their individual members. In the absence of an explicit contrary agreement no one can be introduced as a partner without the consent of all other members of the firm (Prime and Scanlan, 1995: 178), and if anyone retires from a partnership at will, this dissolves the firm (Prime and Scanlan, 1995: 276). Partnership agreements which entitle an outgoing member to choose a successor, although legally possible, are rarely adopted in practice (Banks, 1990: 229). Parallel default rules on turnover among partners have been legislated in Australia and New Zealand (Graw, 1996), Canada (VanDuzer, 1997: ch. 2), and France and Germany (Banks, 1990: 21-25).

Under the Uniform Partnership Act (UPA), the legislation governing partnerships in almost all U.S. states, until recently the firm was required to dissolve upon withdrawal by any individual (Hynes, 1995; Hillman 1995). Indeed, the UPA did not allow partners to waive their right to bring about dissolution. Revisions in the 1990s sought to provide more stability by substituting an ‘entity’ theory of partnership under which a business unit can survive the departure of individual members. But the default procedure is still for the partnership to dissolve upon withdrawal by a single partner. This ensures that continuing members hold veto rights over the choice of a successor.

Similar policies are found in worker cooperatives or labor-managed firms. Such firms seldom treat a membership position as the private property of an individual worker or permit members to sell their positions in the firm to outsiders. One well-known exception involves the plywood cooperatives of Oregon and Washington (Craig and Pencavel, 1992; Pencavel and Craig, 1994; Pencavel, 2002). But even here, a majority of worker-owners must give prior approval before a membership position can be sold to a new worker.

The most obvious reason why partners might insist upon unanimous consent, or at least a majority vote, before accepting a new colleague is that they are concerned with the quality of the candidate. Applicants will normally vary in their skill, judgment, motivation, risk tolerance, and collegiality among other things. For enterprises in which net income is shared, decision-making is joint, and personal liability is unlimited, it is unsurprising that incumbents pay considerable attention to the characteristics of new applicants. To grasp why free transferability of partnership positions is rare, an academic economist need only imagine a system in which colleagues can sell their professorships to the highest bidder.

Allowing membership transactions to take place on an open market often imposes a negative externality on the partners who stay behind, because the departing member finds it attractive to sell out to a low-quality successor. If those remaining behind would not have made the same decision, the joint payoff of the parties is reduced. In principle this problem could be eliminated by having continuing members bribe departing members to recruit their successors differently, but if it is costly to negotiate an agreement of this sort then partners will prefer a system in which insiders retain control over vacancies.

Such arrangements are less necessary when individuals contribute cash or property to an enterprise, rather than labor services, on the reasonable assumption that informational asymmetries matter less for non-human inputs. A key case in point is the use of limited partnerships in the real estate, oil and gas, cable TV, and equipment leasing industries. An active secondary market for such partnership positions has developed in the U.S. since the 1980s (Wollack and Donaldson, 1992; Denning and Shastri, 1993; Allen, 1995; Barber, 1996). Limited liability partners do not engage in operational activities, lack voting rights, and are viewed by the tax authorities as 'passive' investors. Adverse selection issues thus appear negligible as compared with firms in which the partners supply professional skills or participate in business decisions. Conversely, although closely held corporations enjoy limited liability, share transactions often require prior approval from the other owners due to the prominent role of each individual shareholder in managerial decision-making.

There is empirical evidence that adverse selection problems do arise for professional partnerships. Spurr (1987) argues that there are large quality differences among lawyers even after the signalling effects of grades and school quality are taken into account, and that data on law firms support an adverse selection view of the promotion process. O'Flaherty and Siow (1995) find that screening during the associate phase of a law career is relatively imprecise and that mistakes are costly: they estimate that about half of the people promoted to partner by large New York law firms are unqualified and should leave. Landers, Rebitzer, and Taylor (1996) present survey evidence for law firms which indicates that promotion decisions are responsive to hours worked in a manner consistent with adverse selection, but not with predictions from agency theory.

The framework developed here can be viewed as a special case within a larger class of models where property rights over vacancies are assigned to continuing partners in order to avoid the externality problems associated with replacement workers. However, there are several reasons to focus more specifically on adverse selection. First, such models provide detailed predictions about the nature of market equilibrium (separation versus pooling, for example), and these details are important in determining whether or not property rights over vacancies affect the joint payoff of the partners. Second, as mentioned above, the evidence suggests that adverse selection in fact arises for partnerships. Finally, the adverse selection approach has interesting welfare implications: for instance, some equilibria always reduce total surplus if the existing allocation of agents to firms is not already surplus-minimizing.

Section 2 lays out the basic model. I assume there are two types of worker, high productivity (type-a) and low productivity (type-b). Partnerships consist of two workers who share profits equally. Vacancies are filled by having incumbents announce prices for admission to their firms. Section 3 characterizes price equilibria for given fractions of type-a and type-b workers on each side of the market. Three equilibria can arise: a separating equilibrium in which high- and low-quality incumbents announce different prices for entry to the firm; a pooling equilibrium in which incumbents of both types choose the same price and unattached agents of both types apply for membership; and another pooling equilibrium in which all incumbents announce the same price but only low-quality types apply.

Section 4 identifies conditions under which control over vacancies by continuing workers dominates control by departing workers. Departing workers sell out to whoever will pay the most, and due to adverse selection this is a low-quality agent. If the continuing worker would have done the same, as is sometimes true, property rights are irrelevant. But if this incumbent would have chosen a low entry price to attract high-quality applicants, it is necessary to bribe the departing worker to make the same choice. Such side payments are likely to entail bargaining and enforcement costs, which can be avoided by committing the firm in advance to a system where continuing workers are responsible for filling vacancies.

Section 5 shows that market transactions always reduce total surplus when there is a separating equilibrium, or a pooling equilibrium in which only low-quality types apply. From the standpoint of surplus maximization, in either of these cases it would be preferable to shut down the market and preserve the existing allocation of workers to firms, whatever it may be. When there is a pooling equilibrium in which both types apply, market sorting may either raise or lower total surplus relative to the initial allocation.

Section 6 relaxes some assumptions and discusses ways in which adverse selection can be alleviated in practice. Proofs are provided in an appendix.

2. Description of the Model

There are two kinds of worker: type-a (high quality) and type-b (low quality). All workers are risk neutral. Production in a partnership firm involves two workers who share profits equally. Such sharing rules are used in various partnerships and cooperatives, and arise naturally when individual productivity is unverifiable both before and after production occurs. I assume that at least one worker must have prior experience in the same firm due to a need for specific skills. The required skills are readily transmitted on the job from an incumbent to a novice, but two incumbents cannot profitably sell the firm to two novices. These assumptions will be reconsidered in section 6.

Per-person income (g) in a partnership depends symmetrically on types, with

$$g_{aa} > g_{ab} = g_{ba} > g_{bb} > 0.$$

The per-person marginal product of worker quality when one worker is fixed at type-b is $\Delta g_b \equiv g_{ab} - g_{bb}$. The marginal product of quality when one worker is fixed at type-a is $\Delta g_a \equiv g_{aa} - g_{ab}$. I assume $\Delta g_a > \Delta g_b$ throughout. For example, a talented lawyer's advice about difficult cases will generally raise the productivity of colleagues, and this effect is likely to be stronger when these colleagues are also highly talented. Such complementarities occur often in team production tasks with a division of labor, and under full information tend to promote formation of firms in which the partners have similar ability (Sherstyuk, 1998).

The workers have outside options worth $r_a > 0$ and $r_b > 0$ which are interpreted as payoffs from self-employment or home production. I assume $\Delta r \equiv r_a - r_b > 0$ so better partners have better outside options. Thus a lawyer who is unusually productive in a team is also unusually productive when self-employed. Moreover, $g_{ab} > r_a$ and $g_{bb} > r_b$ so each type wants to join a partnership even if the match is certain to involve a type-b colleague. The higher return from a partnership position is due to the quasi-rent associated with earlier investments by these firms in physical assets, proprietary information, client contacts, or brand-name reputation (Carr and Mathewson, 1990).

The number of type-a agents in the population is n_a and similarly for n_b, with $n_a + n_b = n$. The number of partnership firms is m. The fixed number of firms may reflect a short-run situation in which physical or human capital is fixed. Alternatively, the number of firms may be set through a long-run zero profit condition in the product market, where entry requires a sunk set-up cost (perhaps an initial investment in firm-specific skills by the founding partners, or investments of the sort described in the last paragraph). These issues are not directly relevant to present concerns and will not be modeled. I assume

<u>A1</u> $\quad n_a \geq 2m \quad$ and $\quad n_b \geq 2m$

Thus it is feasible to staff all partnership firms entirely with high-quality workers or entirely with low-quality workers. The idea behind A1 is that there are many more candidates for partnership firms than available positions. Workers who are not currently partners pursue solo careers but are available for recruitment if a vacancy arises. These workers are called 'unattached'. I often denote incumbents by the upper-case letters A or B and use the lower-case a or b for unattached workers. The parameters n_a, n_b, and m are taken to be large.

I also assume throughout that

$\underline{A2}$ $$2\Delta g_b > \Delta r > \Delta g_a > \Delta g_b > 0$$

The last two inequalities have already been discussed. The inequality $2\Delta g_b > \Delta r$ implies that AB firms provide more total surplus than BB, and together with $\Delta g_a > \Delta g_b$ implies that AA firms provide more surplus than AB. Hence under full information all matches would be AA. The inequality $\Delta r > \Delta g_a$ says that the income gap between good and bad workers is always greater in self-employment than in partnerships. This is plausible for two reasons. First, the direct productivity benefit from high ability must be shared in a partnership but not in a proprietorship (g is a per-person payoff). Second, productivity differentials are likely to be muted in a partnership if income sharing leads to lower effort by both types.

The analytic consequence is that any partnership offer which attracts type-a workers attracts type-b as well, so there is no equilibrium in which only high-quality workers apply. To formalize this, let $s_a(p,\theta) = \theta g_{aa} + (1-\theta)g_{ab} - r_a - p$ be the surplus of an unattached type-a worker from a match with an incumbent who is thought to be type-A with probability θ, given an admission price of p. Let $s_b(p,\theta) = \theta g_{ab} + (1-\theta)g_{bb} - r_b - p$ be the corresponding surplus for a type-b worker. A2 implies $s_b(p,\theta) > s_a(p,\theta)$ for all p and θ. Thus any match giving non-negative surplus to unattached type-a workers gives positive surplus to type-b.

The sequence of events runs as follows. Initially there is some distribution of firm types $\pi = (\pi_{aa}, \pi_{ab}, \pi_{bb})$, where π_{aa} is the fraction of firms with two type-a incumbents and so on. Partners know each other's type but not the type of anyone else. In each firm, the members end their current match if a larger joint payoff can be obtained through separation. Partnerships may also dissolve exogenously. In either case, the partners themselves decide who will keep the firm's assets and become the incumbent in a successor firm (the effects of constraining this decision will be addressed in section 6).

After separations occur, a market opens for partnership positions. Earlier events have determined the number of incumbents and unattached agents of each type. Incumbent partners who have vacancies announce prices at which new partners can join. This reflects the relative scarcity of partnership positions and implies that demand-side agents capture all surplus, subject to informational constraints. Workers are matched to vacancies through a rationing scheme to be described in section 3. Those workers who are unattached after the market closes get their outside options, as do incumbents with unfilled vacancies. Partners in firms where both positions are occupied receive the incomes appropriate to their types.

3. Price Determination

This section examines the equilibria that can arise after workers have been assigned to the demand and supply sides of the market. Let λ be the fraction of demand-side agents (incumbents with vacancies) who are type-a, and let μ be the fraction of supply-side agents (unattached workers) who are type-a. These fractions are parametric for individual agents.

Positions are allocated using the following rationing scheme. First, each incumbent simultaneously announces a price p at which a new partner can join. Next, each unattached worker simultaneously decides whether to apply at the firms announcing the lowest price. Multiple applications are allowed, and workers apply to a vacancy in case of indifference. If there are fewer firms than total applicants at the lowest price, all vacancies at this price are filled by matching a randomly chosen subset of applicants to firms. On the other hand if more firms have announced the lowest price than there are applicants at this price, a randomly chosen subset of vacancies is filled. If there are no applicants, the vacancies of the lowest-price firms remain unoccupied. This procedure is repeated for the firms having the next highest price and so on, until the highest-price firms are reached.

Unattached workers assign probability $\theta(p)$ to the event that an incumbent choosing price p is type A. In a separating equilibrium all type-A incumbents charge some price p_A and all type-B incumbents charge a different price $p_B \neq p_A$, so equilibrium requires $\theta(p_A) = 1$ and $\theta(p_B) = 0$. In a pooling equilibrium both incumbent types announce the same price p* and beliefs must satisfy $\theta(p^*) = \lambda$. I denote a separating equilibrium by SE, a pooling equilibrium in which both types of unattached worker apply by PE_{ab}, and a pooling equilibrium in which only type-b applies by PE_b.

Proposition 1. Define the following prices:

$p_b^\lambda = \lambda g_{ab} + (1-\lambda)g_{bb} - r_b$ (zero surplus for type-b with beliefs $\theta = \lambda$)

$p_b^0 = g_{bb} - r_b$ (zero surplus for type-b with beliefs $\theta = 0$)

$p_a^\lambda = \lambda g_{aa} + (1-\lambda)g_{ab} - r_a$ (zero surplus for type-a with beliefs $\theta = \lambda$)

$p_c = p_b^0 - \mu\Delta g_b$ (B is indifferent between an all-b applicant pool at price p_b^0 and a mixed a,b applicant pool at price p_c)

Assume that prices favorable to incumbents are selected whenever multiple equilibria exist within the same class. The only separating and pooling equilibria are as follows.

(i) SE exists iff $g_{ab} - r_a \leq p_c$. The prices are $p_A = p_c$ and $p_B = p_b^0$. SE is supportable by the beliefs $\theta(p) = 1$ for $p \leq p_c$ and $\theta(p) = 0$ otherwise.

(ii) PE_{ab} exists iff $p_c \leq p_a^\lambda$. The price is p_a^λ. PE_{ab} is supportable by the beliefs $\theta(p) = \lambda$ for $p \leq p_a^\lambda$ and $\theta(p) = 0$ otherwise.

(iii) Assume $\theta(p)$ is non-increasing in p. PE_b exists iff $p_a^\lambda \leq p_b^\lambda - \mu\Delta g_a$. The price is p_b^λ. PE_b is supportable by the beliefs $\theta(p) = \lambda$ for $p \leq p_b^\lambda$ and $\theta(p) = 0$ otherwise.

Remark 1. The only reason why an incumbent might announce a lower price is to attract a better applicant pool. Due to the complementarity condition $\Delta g_a > \Delta g_b$ in A2, an improved applicant pool is always more beneficial to type-A incumbents than to type-B. Unattached agents should thus give at least as much probability to A at lower prices. The constraint that $\theta(p)$ be non-increasing does not bind for SE or PE_{ab} since if these equilibria exist at all, they can always be supported by such beliefs. But as will be explained later, this constraint does restrict the subset of the (λ, μ) parameter space in which PE_b exists.

Remark 2. In every equilibrium all vacancies are filled. At every (λ, μ) $[0,1]^2$ SE exists, PE_{ab} exists, or both. Total surplus is highest for PE_{ab}, intermediate for SE, and lowest for PE_b whenever the relevant equilibria exist.

Remark 3. In SE successful applicants of both types get rents at the low price p_A; there is no rent for successful applicants (who are type-b only) at the high price p_B. In PE_{ab} successful applicants of type-b receive a rent while type-a applicants do not. In PE_b there is no rent for successful applicants (who are type-b only). If SE and PE_{ab} both exist, incumbents of both types prefer PE_{ab}. If PE_{ab} and PE_b both exist, incumbents of both types prefer PE_b. If SE and PE_b both exist, type-B incumbents prefer PE_b; type-A incumbents prefer PE_b when $\mu(\Delta g_a - \Delta g_b) < \lambda\Delta g_b$ but prefer SE when the inequality is reversed.

The equilibria arising for various regions of (λ, μ) space are shown in Figure 1. To gain some intuition about these results, first focus on the region in which SE exists: that is, $\mu \leq (\Delta r - \Delta g_b)/\Delta g_b$. For the moment consider only PE_{ab} and SE. At a fixed value of μ, if incumbents are mostly low quality then λ is small. A pooling equilibrium PE_{ab} cannot exist in this case because the small fraction of A incumbents means that the associated price p_a^λ

would be low. Incumbents of type B, who by complementarity are less concerned with applicant quality than type A, prefer to deviate upward to the price p_b^0, attract only type-b applicants, and extract all of the surplus from these applicants.

When there are enough high-quality incumbents the pooling price p_a^λ will be higher and neither A nor B will find it profitable to deviate in this way. PE_{ab} therefore exists. But SE also exists because if applicants have sufficiently pessimistic beliefs a type-B incumbent charging the high price $p_B = p_b^0$ in SE cannot attract type-a workers without cutting price to p_A (which is below the pooling price p_a^λ). Given such beliefs, B prefers to remain at p_B.

When $\mu > (\Delta r - \Delta g_b)/\Delta g_b$ there is no SE. At large μ values more unattached agents are of high quality, which implies that in order to preserve separation p_A must be decreased to prevent B from imitating A and attracting both types. Eventually p_A becomes so low that even type-a workers with very pessimistic beliefs can obtain positive surplus by applying at p_A. This tempts B to cut price from p_B to a level slightly above p_A, destroying separation.

Assume beliefs are non-increasing as in Remark 1. Then PE_b exists only at low μ values as shown in Figure 1, because at high μ values an A incumbent is tempted to reduce price and attract high-quality applicants. The region of PE_b existence expands if applicants hold pessimistic beliefs when they see a price reduction so that A is less able to attract type-a workers by deviating in this way. Maximally pessimistic beliefs yield a boundary for PE_b existence in Figure 1 with the same left vertical intercept but a positive slope of $\Delta g_b/\Delta g_a$.

4. Property Rights

Vacancies can arise in partnerships for various reasons. Section 5 will show that it is sometimes mutually advantageous for partners to separate because a greater joint payoff can be obtained when one partner exits to the market while the other recruits a replacement. Separations can also occur exogenously. As discussed in section 1, it is standard practice that when a vacancy arises, the continuing partner sets the terms on which a newcomer can join. Section 3 assumed that continuing partners 'own' vacancies in this sense.

One possible explanation for this pattern runs as follows. A continuing incumbent internalizes the tradeoff between a higher price for admission and a lower-quality applicant pool. Maximization of the continuing incumbent's payoff automatically maximizes the joint payoff of the partners, because the price at which a replacement worker is recruited has no effect on the return a departing worker can obtain from the external market. Therefore, side payments at the time of exit are unnecessary under this system. But the departing worker is indifferent toward the quality of a replacement, and in the absence of side payments would sell out for the highest price at which someone will apply. This implies that a low-quality worker is always recruited. This problem can be solved by having the continuing member bribe the departing member to set a lower price. But if such deals are costly to negotiate then it is better to avoid these prospective costs when the firm is established, by committing to a system in which the continuing member controls the vacancy when separation occurs.

There are, however, several qualifications to this story. First, if the fixed cost of arranging a bribe to restore joint maximization is too large then no bribes will be paid when departing members control vacancies. If the objectives of these agents deviate from joint maximization, firms will use different strategies for admitting members depending on who chooses the admission price. Hence unattached agents will need to condition their beliefs $\theta(p)$ on the property rights structure of a firm because prices set by continuing incumbents may be informative, while prices set by departing members generally will not be.

A further complication is that reassigning control over vacancies could influence the incentives of partners to dissolve their matches. If this effect differed across firms having different incumbent types, again unattached agents would need to take property rights into account in forming their beliefs. Such issues are ignored here because Proposition 2 below applies to situations where bribes maintain joint maximization regardless of property rights. Section 5 will assume that voluntary separations maximize the joint payoff of the parties.

Even if firms with different property rights systems adopt the same pricing strategy, the nature of the equilibrium also matters. If the continuing member would have chosen a high entry price and recruited a low-quality worker, there is no need to use bribes to restore joint maximization because the departer makes the same decision. But in some equilibria these two agents choose differently and therefore bribes become important. The continuing member's type can be relevant as well. In a separating equilibrium, for example, high- and low-quality incumbents announce distinct prices. As a result, type-A incumbents will bribe the departer if the associated cost is low enough, but type-B incumbents will not.

For these reasons, it is not trivial to show that continuing partners will set the terms on which vacancies are filled. Proposition 2 states sufficient conditions for this to be true, where these conditions vary with the prevailing market equilibrium (PE_{ab}, SE, or PE_b).

Proposition 2.

(i) Suppose the market equilibrium is PE_{ab} and unattached agents form beliefs $\theta(p)$ as described for PE_{ab} in Proposition 1, regardless of the property rights over vacancies in a given firm. These beliefs are justified (every firm maximizes the joint payoff of its partners when separation occurs, regardless of property rights) if the fixed cost $\varepsilon > 0$ to the continuing member of bribing the departing member satisfies $\varepsilon < \varepsilon(\lambda,\mu) \equiv (1+\mu)\Delta g_b + \lambda\Delta g_a - \Delta r$, where $\varepsilon(\lambda,\mu) > 0$ above the downward-sloping line bounding the PE_{ab} existence region in Figure 1 and $\varepsilon(\lambda,\mu) \to 0$ as (λ,μ) approaches this boundary. If $\varepsilon(\lambda,\mu) > \varepsilon > 0$ and a separation has positive probability, the joint payoff of the partners is maximized at the firm formation stage by giving control over vacancies to the continuing member.

(ii) Suppose the market equilibrium is SE and unattached agents form beliefs $\theta(p)$ as described for SE in Proposition 1, regardless of the property rights over vacancies in a given firm. These beliefs are justified (every firm maximizes the joint payoff of its partners when separation occurs, regardless of property rights) if the fixed cost $\varepsilon > 0$ to the continuing member of bribing the departing member satisfies $\varepsilon < \varepsilon(\mu) \equiv \mu(\Delta g_a - \Delta g_b)$, where $\varepsilon(\mu) > 0$ above the horizontal axis in Figure 1 and $\varepsilon(\mu) \rightarrow 0$ as μ approaches this boundary. If $\varepsilon(\mu) > \varepsilon > 0$ and a separation resulting in a type-A incumbent has positive probability, the joint payoff of the partners is maximized at the firm formation stage by giving control over vacancies to the continuing member.

(iii) Suppose the market equilibrium is PE_b and unattached agents form beliefs $\theta(p)$ as described for PE_b in Proposition 1, regardless of the property rights over vacancies in a given firm. These beliefs are justified because every firm maximizes the joint payoff of its partners when separation occurs, regardless of property rights.

Joint payoff maximization occurs automatically when the continuing incumbent has the property rights, as was assumed in Proposition 1. In the PE_{ab} case, restoration of joint maximization under departer rights requires that bribes be paid by both types of incumbent. The condition involving $\varepsilon(\lambda, \mu)$ ensures that this occurs. In the SE case, no bribe is needed if the continuing incumbent is type-B, but a bribe must be paid if the incumbent is type-A. Because outsiders cannot observe the incumbent's type, the condition involving $\varepsilon(\mu)$ must be satisfied in both cases if the beliefs $\theta(p)$ are to be justified. For PE_b it does not matter who has control over vacancies or what the incumbent's type is, because continuing and departing members always charge the same price.

5. Welfare Analysis

Assume all firms assign control over vacancies to continuing incumbents, so there is no need for side payments to maintain joint maximization. The analysis of equilibrium in section 3 therefore applies. This section considers the incentives of partners to preserve or dissolve their match, along with the implications of these decisions for aggregate surplus.

If the parties want to preserve a match, it remains intact with probability α (0,1) but ends involuntarily with probability 1-α. This could occur, for example, if one partner is obliged to relocate. Matches end voluntarily whenever the sum of the payoffs of their members can be increased by having one partner leave for the market while the other recruits a replacement. I assume that the non-human assets lending continuity to the firm (such as client lists or brand names) can be inherited by either person, so the partners can choose their roles in a joint-maximizing manner regardless of the reasons for separation. Section 6 considers the consequences of constraining these role assignments.

Let ρ_{aa} $[0,\alpha]$ be the probability that an AA match remains intact. If the partners have a higher joint payoff from staying together than from separation, $\rho_{aa} = \alpha$ because all separations are involuntary. If the partners have a higher joint payoff from separation, ρ_{aa} = 0. If separation and continuation provide identical joint payoffs then any ρ_{aa} $[0,\alpha]$ is optimal. The probabilities ρ_{ab} and ρ_{bb} are defined similarly. For AB matches let η [0, 1] be the (optimal) probability that the A partner becomes the incumbent after a voluntary or involuntary separation, with 1-η the corresponding probability for the B partner.

Let $\pi = (\pi_{aa}, \pi_{ab}, \pi_{bb})$ be some historically given distribution of firm types. Market transactions update π to give a final distribution $\pi' = (\pi_{aa}', \pi_{ab}', \pi_{bb}')$ according to

$$\pi_{aa}' = [\rho_{aa} + (1-\rho_{aa})\mu_A]\pi_{aa} + (1-\rho_{ab})\eta\mu_A\pi_{ab}$$

$$\pi_{ab}' = (1-\rho_{aa})(1-\mu_A)\pi_{aa} + \{\rho_{ab} + (1-\rho_{ab})[\eta(1-\mu_A) + (1-\eta)\mu_B]\}\pi_{ab} + (1-\rho_{bb})\mu_B\pi_{bb}$$

$$\pi_{bb}' = (1-\rho_{ab})(1-\eta)(1-\mu_B)\pi_{ab} + [\rho_{bb} + (1-\rho_{bb})(1-\mu_B)]\pi_{bb}$$

where μ_A is the probability that a type-A incumbent is matched with a type-a applicant, and μ_B is the probability that a type-B incumbent is matched with a type-a applicant. These probabilities depend upon the prevailing market equilibrium (PE_{ab}, SE, PE_b), as explained below. The transition equations will be abbreviated $\pi' = Q\pi$ where Q is a 3x3 matrix.

Recall that λ is the ratio of type-A incumbents with vacancies to the total number of incumbents with vacancies to fill, and μ is the ratio of unattached type-a workers to the total number of unattached workers. Both are computed starting from the initial distribution of firm types π, taking into account voluntary and involuntary separations. This gives

$$\lambda = [(1-\rho_{aa})\pi_{aa} + \eta(1-\rho_{ab})\pi_{ab}] / [(1-\rho_{aa})\pi_{aa} + (1-\rho_{ab})\pi_{ab} + (1-\rho_{bb})\pi_{bb}]$$

$$\mu = \{n_a - (1+\rho_{aa})\pi_{aa}m - [\rho_{ab}+\eta(1-\rho_{ab})]\pi_{ab}m\}/\{n - m[(1+\rho_{aa})\pi_{aa} + (1+\rho_{ab})\pi_{ab} + (1+\rho_{bb})\pi_{bb}]\}$$

where $0 \leq \lambda \leq 1$ and $0 < \mu < 1$. The fact that μ is positive and less than unity follows from A1 and the relations $\rho_{aa} \leq \alpha < 1$, $\rho_{ab} \leq \alpha < 1$, $\rho_{bb} \leq \alpha < 1$. PE_{ab} implies $\mu_A = \mu_B = \mu$ in the matrix Q. SE implies $\mu_A = \mu$ but $\mu_B = 0$ because unattached type-a workers do not apply at the high price p_B. PE_b implies $\mu_A = \mu_B = 0$ because type-a workers never apply.

Surplus per firm at the initial distribution π and final distribution π' is

$$W = \pi_{aa}(2g_{aa} - 2r_a) + \pi_{ab}(2g_{ab} - r_a - r_b) + \pi_{bb}(2g_{bb} - 2r_b)$$

$$W' = \pi_{aa}'(2g_{aa} - 2r_a) + \pi_{ab}'(2g_{ab} - r_a - r_b) + \pi_{bb}'(2g_{bb} - 2r_b)$$

where π' is computed from $\pi' = Q\pi$ as above. Because the number of firms is constant the effect of market transactions on total surplus is indicated by $\Delta W \equiv W' - W$, where

$$\Delta W = (2\Delta g_a - \Delta r)[-(1-\rho_{aa})(1-\mu_A)\pi_{aa} + (1-\rho_{ab})\eta\mu_A\pi_{ab}]$$
$$+ (2\Delta g_b - \Delta r)[(1-\rho_{bb})\mu_B\pi_{bb} - (1-\rho_{ab})(1-\eta)(1-\mu_B)\pi_{ab}]$$

<u>Proposition 3</u>.

(i) All equilibria have $\rho_{aa} = \alpha$ and $\eta = 0$.

(ii) PE_{ab} always has $\rho_{bb} = 0$. If $\pi_{aa} > 0$ then SE and PE_b also have $\rho_{bb} = 0$.

(iii) All equilibria have $\pi_{aa}' < \pi_{aa}$ whenever $\pi_{aa} > 0$. If $\pi_{aa} > 0$ and $\pi_{ab} > 0$ both hold then SE and PE_b have $\pi_{bb}' > \pi_{bb}$. SE and PE_b have $\Delta W < 0$ whenever $\pi_{bb} < 1$.

(iv) Suppose n_a and n_b are large relative to m so that μ can be treated as exogenous, and the initial distribution π is generated by randomly assigning workers to firms so that $\pi_{aa} = \mu^2$, $\pi_{ab} = 2\mu(1-\mu)$, and $\pi_{bb} = (1-\mu)^2$. Suppose further that PE_{ab} occurs at the relevant μ values. Then if $\alpha > 1/2$, there is some μ^* (0,1) such that $\Delta W > 0$ for $\mu < \mu^*$ and $\Delta W < 0$ for $\mu > \mu^*$; while if $\alpha \leq 1/2$, $\Delta W < 0$ holds for all μ (0,1).

Proposition 3 yields the following results. AA matches never dissolve voluntarily ($\rho_{aa} = \alpha$), while in general BB matches do ($\rho_{bb} = 0$). AB matches may or may not dissolve voluntarily, but the B partner is always the incumbent in the successor firm if a separation occurs ($\eta = 0$) . This may seem surprising in light of A's productivity advantage within the firm, and the fact that B would receive a rent under PE_{ab} if successfully matched elsewhere while A would not. But A1 imposes an upper bound on the probability that the departing worker will be matched upon leaving the firm. Furthermore, A2 implies that A's superior reservation alternative outweighs the productivity benefit from keeping A in the firm. The partners are therefore better off sending A to the market and having B seek a replacement.

Since any AB or BB split leaves a B incumbent, the successor firm can only have AB or BB. Although AA partnerships never split voluntarily, separations sometimes occur for exogenous reasons. When this happens there is positive probability that the next match will be AB because there are always some unattached type-b workers on the supply side. This reduces the share of AA firms in the population ($\pi_{aa}' < \pi_{aa}$).

Assuming $\pi_{aa} > 0$, BB matches dissolve in all equilibria. Both SE and PE_b have the feature that B incumbents can only be matched with type-b applicants ($\mu_B = 0$), so all of the dissolving BB pairs are reconstituted as BB. If $\pi_{ab} > 0$ holds then in these equilibria the fraction of BB firms increases ($\pi_{bb}' > \pi_{bb}$), because AB separations lead to B incumbents ($\eta = 0$), and B incumbents are never matched with type-a applicants ($\mu_B = 0$). SE and PE_b imply $\Delta W < 0$ because in each case some AA pairs are converted to AB and some AB pairs are converted to BB, but movements in the opposite direction are impossible. The only exception occurs when $\pi_{bb} = 1$, which implies that total surplus is already minimized.

The welfare effects of PE_{ab} are more complex. A useful simplification is to assume that workers of both types are numerous relative to firms so $\mu = n_a/n$ is fixed, and to use a random initial assignment of workers to firms as a benchmark. The results are shown in Figure 2. If μ is small AB pairs never dissolve voluntarily and $\rho_{ab} = \alpha$. At intermediate values PE_{ab} has $0 < \rho_{ab} < \alpha$ and ρ_{ab} is a decreasing function of μ. If μ is large AB pairs always dissolve and $\rho_{ab} = 0$. The other locus in Figure 2 indicates the minimum value of ρ_{ab} consistent with $\Delta W \geq 0$. If $\alpha > 1/2$ the two curves intersect at some unique μ^* as in Figure 2, so $\Delta W > 0$ for $\mu < \mu^*$ and $\Delta W < 0$ for $\mu > \mu^*$. But if $\alpha \leq 1/2$ then $\Delta W < 0$ for all $\mu > 0$ and market sorting is inferior to randomization.

A caveat about PE_{ab} existence is needed because when the initial firm distribution is chosen randomly, $\mu \to 0$ implies $\lambda \to 0$. Figure 1 establishes that PE_{ab} cannot occur in this situation so there is no possibility of increasing welfare at small values of μ; either SE or PE_b must occur and both decrease total surplus. Whether PE_{ab} existence and increased total surplus coincide for some range of μ values depends on the parameters of the model. However, this is not impossible. It can be shown that PE_{ab} always exists for $\mu \geq \mu_0$ in Figure 2 and that the $\Delta W = 0$ curve intersects the declining part of the equilibrium ρ_{ab} locus when $\alpha > 3/4$ and $\Delta g_a \approx \Delta g_b$. Thus in this case $\Delta W > 0$ for μ $[\mu_0, \mu^*)$.

6. Extensions

One important assumption in section 5 was that partners who separate can choose who will be the incumbent in the successor firm, even if the separation was involuntary. It is easy to imagine cases where this might not be true. For example, a partner who moves for family reasons may be unable to take the firm's clients to a new location.

Assume optimistically that the probability of an A incumbent following an AB split is $\eta = 1/2$ whether the separation is voluntary or not. If μ is exogenous and workers are initially matched randomly, it can be shown that a separating equilibrium still reduces total surplus. In a pooling equilibrium where only type-b applies, the result is stronger: even if the initial firm distribution π is arbitrary, surplus falls for any value of the probability η. This occurs despite the fact that AB separations may result in A incumbents, because under PE_b such incumbents are never matched to type-a workers ($\mu_A = 0$). On a brighter note, a positive value of η does make welfare gains more likely under PE_{ab}.

I have assumed throughout that firms must include one experienced partner because firm-specific skills must be transmitted to newcomers. This prevents insiders from selling the firm to two outsiders simultaneously. It also implies that randomly assigning workers to firms may not be feasible unless this occurs before the specific skills have been acquired.

Suppose incumbents can sell firms to pairs of unattached workers after the ordinary partnership market closes. When $\mu < (\Delta r - \Delta g_b)/\Delta g_a$ the highest price unattached workers will pay is $g_{bb} - r_b$ and only type-b applies. Otherwise the highest price is $\mu g_{aa} + (1-\mu)g_{ab} - r_a$ and both types apply. In the former case it is never profitable for incumbents to sell the firm, so this extension does not overturn any results from section 5.

When $\mu > (\Delta r - \Delta g_b)/\Delta g_a$ a BB pair always sells out, AB may sell out if μ is high enough, and AA never does. If AB does not sell out there is a clear surplus gain relative to any starting distribution because some BB matches are converted to AA or AB. If AB does sell out, the net change in welfare is $\Delta W = \pi_{aa}' s_{aa} + (1-\pi_{aa}')W_\mu - W'$ where π_{aa}' is the share of AA firms after the ordinary partnership market closes as in section 5, s_{aa} is the surplus from an AA match, W_μ is total surplus from a random assignment of workers according to μ, and W' is total surplus after the partnership market closes. Since $s_{aa} \geq W_\mu$ it follows that if the partnership market depresses welfare below the random assignment level ($W_\mu > W'$) there is a welfare gain from permitting incumbents to sell out afterward.

One could argue that in practice adverse selection problems are mitigated by having high-quality applicants and incumbents signal their true type. Incumbents can also screen members, for instance by requiring a probationary period before promotion to full partner. But signalling and screening are costly and do not entirely remove information asymmetries even when used. Academic records, for example, shed little light on traits such as honesty or cooperativeness. As noted in section 1, there is good evidence that adverse selection is a significant problem for professional partnerships despite their use of such mechanisms.

Another potential solution is for incumbents to give low-productivity newcomers a lower profit share ex post. But if productivity is unverifiable high-productivity newcomers might receive similar treatment. Promotion tournaments restrain moral hazard problems of this sort, but have poor incentive features when cooperation among the younger partners is desired. Perhaps the best solution is for incumbent partners to build a reputation for high productivity and fairness toward newcomers. The feasibility of this will undoubtedly vary.

One final way to alleviate adverse selection problems is through regulation. Leland (1979) argues that quality standards imposed by professional associations can be justified in this way, a point of view contested by Ryoo (1996). Even when such quality regulation exists it is likely to be a coarse filter which only screens out the worst practitioners, leaving ample scope for adverse selection among the remaining population.

Appendix

Proof of Proposition 1

Lemma 1. There are enough unattached workers of each type to fill all vacancies.

This is immediate from A1.

Lemma 2. All vacancies are filled in any pooling or separating equilibrium.

Consider a pooling equilibrium in which no one applies at the equilibrium price p*, so all unattached and incumbent agents get zero surplus. All unattached workers would apply at $p = 0$ regardless of their beliefs because $g_{ab} > r_a$ and $g_{bb} > r_b$. An incumbent who deviates to this price therefore gets positive surplus, a contradiction. Hence at least one type of unattached worker applies at p* and by Lemma 1 this ensures that all vacancies are filled. Next consider a separating equilibrium such that no one applies at one price, but at least one type of unattached agent applies at the other. Those incumbents choosing the former price can switch to the latter and obtain positive surplus, again a contradiction. If neither price attracts applicants the argument is the same as for pooling. Since at least one type applies at each price in a separating equilibrium, all vacancies are filled by Lemma 1.

Lemma 3. Type-b workers apply at both prices in any separating equilibrium.

By Lemma 2 some worker type applies at the higher price. If type-b does not apply then type-a must, implying that type-a workers get non-negative surplus from a successful match at this price. But the surplus for type-b workers at any given p and $\theta(p)$ is larger than the corresponding surplus for type-a, so type-b workers must derive positive surplus from a successful match at the same price. Hence type-b workers apply at the higher price.

Next I show that type-b workers apply at the lower price. Suppose $p_A > p_B$. From Lemma 2 some type applies at p_B. Assume only type-a workers apply at p_B and let μ_A be the equilibrium fraction of workers applying at price p_A who are type-a. It must be true that $\mu_A g_{aa} + (1-\mu_A)g_{ab} + p_A \geq g_{aa} + p_B$ because the left side is what incumbent A obtains in equilibrium, while the right side is what A could get by switching to p_B and recruiting a type-a partner for sure. This gives $p_A - p_B \geq (1-\mu_A)\Delta g_a$. Conversely it must be true that $g_{ab} + p_B \geq \mu_A g_{ab} + (1-\mu_A)g_{bb} + p_A$ where the left side is what incumbent B gets in equilibrium and the right side is the payoff from a deviation to p_A. This gives $(1-\mu_A)\Delta g_b \geq p_A - p_B$. Recall from A2 that $\Delta g_a > \Delta g_b$ and note that $\mu_A < 1$ holds because if there is any incumbent A with a vacancy, some type-b workers apply at the higher price p_A. These facts imply $p_A - p_B \geq (1-\mu_A)\Delta g_a > (1-\mu_A)\Delta g_b \geq p_A - p_B$, a contradiction. Hence type-b workers apply at p_B.

Suppose $p_A < p_B$. An unattached type-b worker would derive more surplus from a match with a type-A incumbent at the price p_A than from a match with a type-B incumbent at the higher price p_B. Furthermore, it has already been shown that type-b workers apply at the higher price so their surplus is non-negative at p_B and strictly positive at p_A. Because a successful match at p_A is better than a match at p_B and an unsuccessful application at p_A does not preclude a subsequent application at p_B, type-b workers apply at p_A.

Lemma 4. There is no separating equilibrium with $p_A > p_B$.

Assume $p_A > p_B$. From Lemma 3 type-b workers apply at both prices. It must be true that type-a workers apply at p_B but not at p_A since otherwise incumbent B can attract applicants with at least the same average quality by deviating to the higher price p_A. Since type-a workers apply at p_B we have $s_a(p_B,0) \geq 0$ which implies $g_{ab} - r_a \geq p_B$; and because type-a workers do not apply at price p_A we have $s_a(p_A,1) < 0$ which implies $g_{aa} - r_a < p_A$.

Combining these gives $p_A - p_B > \Delta g_a$. Now suppose an incumbent B deviates to p_A. This is profitable if $g_{bb} + p_A > \mu_B g_{ab} + (1-\mu_B)g_{bb} + p_B$ where μ_B is the equilibrium fraction of applicants at price p_B who are type-a. This holds when $p_A - p_B > \mu_B \Delta g_b$. But $p_A - p_B > \Delta g_a > \mu_B \Delta g_b$ by A2 so the deviation is profitable. Hence $p_A > p_B$ cannot occur.

Lemma 5. SE exists iff $g_{ab} - r_a \leq p_c$. The prices are $p_A = p_c$ and $p_B = p_b^0$. SE is supportable by the beliefs $\theta(p) = 1$ for $p \leq p_c$ and $\theta(p) = 0$ otherwise.

By Lemma 4 any SE must have $p_A < p_B$. It must be that both types apply at p_A but only type-b applies at p_B since otherwise incumbent A can attract applicants with at least the same average quality by deviating to p_B. Incumbent B chooses $p_B = p_b^0$ because type-b will not apply at p_B if $s_b(p_B,0) < 0$ which contradicts Lemma 3, while if $s_b(p_B,0) > 0$ incumbent B could raise price slightly and still attract type-b agents whatever their beliefs. The result $p_B = p_b^0$ implies that type-b derives no surplus from a match at p_B and thus will apply at any lower price such that surplus would be positive from a successful match.

B is indifferent between attracting only type-b at p_B and attracting both types at $p_B - \mu\Delta g_b$. A is indifferent between attracting only type-b at p_B and attracting both types at $p_B - \mu\Delta g_a$. A necessary and sufficient condition to prevent A from imitating B and vice versa is therefore p_A $[p_B - \mu\Delta g_a, p_B - \mu\Delta g_b]$. A further necessary condition for existence of SE is $s_a(p_B - \mu\Delta g_b,0) \leq 0$, which can equivalently be written as $s_a(p_c,0) \leq 0$ or $g_{ab} - r_a \leq p_c$, since otherwise B could deviate to a price slightly above $p_B - \mu\Delta g_b$, attract both types regardless of their beliefs, and gain relative to p_B. Note that the remaining type-a workers not matched at p_A will not apply at p_B and therefore do apply at the deviant price offered by B if it gives them positive surplus. Type-b workers not matched at p_A have zero surplus at p_B whether successfully matched or not, and thus also apply at the deviant price.

To show sufficiency assume $s_a(p_c,0) \leq 0$, set $p_A = p_c$ and $p_B = p_b^0$, and use beliefs $\theta(p) = 1$ for all $p \leq p_c$ with $\theta(p) = 0$ for all $p > p_c$. Type-b has zero surplus at p_B and thus applies at this high price, while type-a has negative surplus and does not. Both type-a and type-b have positive surplus at p_A so both apply at the lower price. Incumbent B cannot profitably deviate to any price $p' > p_A$ since type-a never applies at such prices and neither type applies at prices above p_B. Incumbent A cannot profitably deviate to a price above p_A because only type-b will apply at prices in the interval $(p_A, p_B]$ and a deviation to p_B is not profitable for A if only type-b applies. Again deviations above p_B can be ignored. Because p_A attracts both types, deviations $p' < p_A$ are unprofitable for A and B if the possibility that only type-a applies at such a price can be excluded. But due to $\theta(p') = 1$, type-b prefers a match at p' to a match at p_A and applies at $p' < p_A$. This completes the proof of sufficiency.

In any other separating equilibrium, A announces a lower price. Since equilibria favoring incumbents are selected when more than one equilibrium of the same class exists, SE has the properties described above.

Lemma 6. There is no pooling equilibrium in which type-b does not apply.

Suppose the contrary. By Lemma 2 type-a must apply so at the equilibrium price p^*, A2 implies $s_b(p^*,\lambda) > s_a(p^*,\lambda) \geq 0$. Hence type-b must also apply at the price p^*.

Lemma 7. PE_{ab} exists iff $p_c \leq p_a^\lambda$. The price is p_a^λ. PE_{ab} is supportable by beliefs $\theta(p) = \lambda$ for $p \leq p_a^\lambda$ and $\theta(p) = 0$ otherwise.

Let the equilibrium price be p* so $\theta(p^*) = \lambda$. Because type-a applies in equilibrium $s_a(p^*,\lambda) \geq 0$, implying $p^* \leq p_a^\lambda$. Suppose $p_a^\lambda < p_c$. Then incumbent B prefers attracting both types at p_c to attracting both types at p* and is indifferent between attracting both types at p_c and attracting only type-b at p_b^0. Type-b workers apply at p_b^0 regardless of beliefs. Thus B can profitably deviate from p* to p_b^0. This shows the necessity of $p_c \leq p_a^\lambda$.

To show sufficiency assume the latter inequality holds, set $p^* = p_a^\lambda$, and use beliefs $\theta(p) = \lambda$ for $p \leq p_a^\lambda$ with $\theta(p) = 0$ for $p > p_a^\lambda$. Both types apply at p_a^λ. Type-a gets zero surplus in equilibrium and will not apply at prices above p_a^λ. Neither A nor B deviates to any $p' > p_a^\lambda$ since p_b^0 is the highest price at which type-b will apply, and due to $p_c \leq p_a^\lambda$ this is no better for either A or B than attracting both types at p_a^λ. Neither A nor B deviates to any $p' < p_a^\lambda$ because both types are already attracted at p_a^λ and due to $\theta(p') = \lambda$ type-b always prefers a match at p′ to one at p_a^λ. It is therefore impossible to attract only type-a at such prices. This completes the proof of sufficiency.

Any other pooling equilibrium in which both types apply must have a price lower than p_a^λ. Since equilibria favoring incumbents are selected when more than one equilibrium of the same class exists, PE_{ab} has the properties described above.

Lemma 8. Assume $\theta(p)$ is non-increasing. PE_b exists iff $p_a^\lambda \leq p_b^\lambda - \mu\Delta g_a$. The price is p_b^λ. PE_b is supportable by the beliefs $\theta(p) = \lambda$ for $p \leq p_b^\lambda$ and $\theta(p) = 0$ otherwise.

Let the equilibrium price be p* so $\theta(p^*) = \lambda$. Because type-b applies in equilibrium $s_b(p^*,\lambda) \geq 0$, which implies $p^* \leq p_b^\lambda$. Furthermore $p_b^0 \leq p^*$ since if $p^* < p_b^0$ incumbents could deviate upward to p_b^0 and continue to attract type-b applicants regardless of beliefs. Incumbent A can profitably deviate downward if type-a incumbents apply at some price above $p^* - \mu\Delta g_a$, whether or not type-b also applies. Since $\theta(p)$ is non-increasing, $\theta(p) \geq \lambda$ for all p $(p^* - \mu\Delta g_a, p^*)$. Type-a will apply at prices slightly above $p^* - \mu\Delta g_a$ unless $s_a(p^* - \mu\Delta g_a, \lambda) \leq 0$. Thus a necessary condition for existence is $p_a^\lambda \leq p^* - \mu\Delta g_a$. Because $p^* \leq p_b^\lambda$ this implies $p_a^\lambda \leq p_b^\lambda - \mu\Delta g_a$.

To show sufficiency assume this last inequality holds, set $p^* = p_b^\lambda$, and use beliefs $\theta(p) = \lambda$ for all $p \leq p_b^\lambda$ with $\theta(p) = 0$ for all $p > p_b^\lambda$. Type-b has zero surplus at p_b^λ and thus applies, while type-a has negative surplus and does not. Neither A nor B will deviate upward because neither type applies at prices above p_b^λ. A downward deviation is only profitable if type-a can be attracted. This cannot occur for any p $(p_b^\lambda - \mu\Delta g_a, p_b^\lambda)$ since $s_a(p_b^\lambda - \mu\Delta g_a, \lambda) \leq 0$. Both A and B prefer p_b^λ with exclusively type-b applicants to a price below $p_b^\lambda - \mu\Delta g_a$ which attracts both types, and at $p_b^\lambda - \mu\Delta g_a$ incumbent B prefers to attract only type-b at p_b^λ while A is indifferent. It is impossible to attract only type-a applicants by downward deviations because type-b has zero surplus in equilibrium and derives positive surplus from a match at any price below p_b^λ. This completes the proof of sufficiency.

Any other pooling equilibrium in which only type-b applies must have a price lower than p_b^λ. Since equilibria favoring incumbents are selected when more than one equilibrium of the same class exists, PE_b has the properties described above.

Proposition 1 follows from Lemmas 4-8. Remark 1 is self-explanatory. The first part of Remark 2 follows from Lemma 2, the second from Figure 1, and the third from A2. Remark 3 follows from the equilibrium payoffs and the existence results in Proposition 1.

Proof of Proposition 2

(i) Consider PE_{ab}. Given the beliefs $\theta(p)$ in Proposition 1, the highest price anyone will pay is p_b^0 and this is what a departing member charges if there are no side payments. The resulting payoff to a continuing incumbent is g_{ib} if the incumbent is type i. Such an incumbent would receive a payoff $\mu g_{ia} + (1-\mu)g_{ib}$ if the departer instead charged p_a^λ. No other prices need be considered, because they attract identical applicant pools but yield less revenue. The most the incumbent is willing to pay to induce the departer to switch prices, net of the fixed cost ε for the bribe, is $\mu g_{ia} + (1-\mu)g_{ib} - g_{ib} - \varepsilon$. The least the departer will accept to switch prices is $p_b^0 - p_a^\lambda$. The former exceeds the latter iff $\mu\Delta g_i + \Delta g_b + \lambda\Delta g_a - \Delta r > \varepsilon$. If this inequality holds for i = b, it holds for i = a. Thus a sufficient condition for joint payoff maximization under departer rights, given $\theta(p)$, is $\varepsilon(\lambda,\mu) \equiv (1+\mu)\Delta g_b + \lambda\Delta g_a - \Delta r > \varepsilon$. The beliefs $\theta(p)$ are justified if this inequality holds because every firm chooses price according to joint payoff maximization, as in Proposition 1, regardless of its property rights system. The inequality $\varepsilon(\lambda,\mu) > 0$ holds iff the existence condition for PE_{ab} in Proposition 1 is satisfied as a strict inequality, with $\varepsilon(\lambda,\mu) = 0$ iff this condition holds with equality. If $\varepsilon(\lambda,\mu) > \varepsilon > 0$ and a separation has positive probability, the fixed cost ε is paid with positive probability when the departing member controls vacancies, but not when the continuing member does.

(ii) Consider SE. Given the beliefs $\theta(p)$ from Proposition 1, the highest price anyone will pay is p_b^0 and this is what a departing member charges if there are no side payments. The resulting payoff to a continuing incumbent is g_{ib} if the incumbent is type i. Such an incumbent would receive a payoff of $\mu g_{ia} + (1-\mu)g_{ib}$ if the departer instead charged p_c. Again no other prices need be considered, because they attract identical applicant pools but yield less revenue. The most an incumbent will pay to induce the departer to switch prices, net of the fixed cost ε of arranging the bribe, is $\mu g_{ia} + (1-\mu)g_{ib} - g_{ib} - \varepsilon$. The minimum the departer will accept is $p_b^0 - p_c$. The former exceeds the latter iff $\mu(\Delta g_i - \Delta g_b) > \varepsilon$. When i = b this inequality is irrelevant because even without a bribe the departer charges the same price the continuing incumbent would charge. When i = a and this inequality holds, joint payoff maximization is restored for a type-a incumbent. Thus a sufficient condition for joint payoff maximization under departer rights, given $\theta(p)$, is $\varepsilon(\mu) \equiv \mu(\Delta g_a - \Delta g_b) > \varepsilon$. The beliefs $\theta(p)$ are justified if this inequality holds because every firm sets price according to joint payoff maximization as in Proposition 1, regardless of its property rights system. When $\varepsilon(\mu) > \varepsilon > 0$ and a separation involving a type-A incumbent has positive probability, the fixed cost ε is paid with positive probability if the departing member controls vacancies, but not if the continuing member does.

(iii) Consider PE_b. Given the beliefs $\theta(p)$ from Proposition 1, the highest price that any applicant will pay for admission is p_b^λ and this is the price a departing member charges. It is also the price a continuing incumbent charges, so every firm sets price according to joint payoff maximization as in Proposition 1, regardless of its property rights system.

Proof of Proposition 3

To prove (i) and (ii) it is convenient to examine each equilibrium individually.

Consider PE_{ab}. The equilibrium probability β_{ab} of a successful match is the ratio of the total number of incumbents with vacancies to the total number of unattached workers: $\beta_{ab} = m[(1-\rho_{aa})\pi_{aa} + (1-\rho_{ab})\pi_{ab} + (1-\rho_{bb})\pi_{bb}]/\{n - m[(1+\rho_{aa})\pi_{aa} + (1+\rho_{ab})\pi_{ab} + (1+\rho_{bb})\pi_{bb}]\}$ where $0 < \beta_{ab} < 1$ by A1. AA pairs receive a total payoff of $2g_{aa}$ by staying together. The joint payoff from separation is $\mu g_{aa} + (1-\mu)g_{ab} + p_a^\lambda + \beta_{ab}[\lambda g_{aa} + (1-\lambda)g_{ab} - p_a^\lambda] + (1-\beta_{ab})r_a$. Staying together is strictly preferred if $2 > \mu + \lambda$ which must be true since $\mu < 1$. Thus $\rho_{aa} = \alpha$. BB pairs receive $2g_{bb}$ by staying together. The joint payoff from separation is $\mu g_{ab} + (1-\mu)g_{bb} + p_a^\lambda + \beta_{ab}[\lambda g_{ab} + (1-\lambda)g_{bb} - p_a^\lambda] + (1-\beta_{ab})r_b$. Separation is strictly preferred if $(1+\mu)\Delta g_b + \lambda\Delta g_a + \beta_{ab}[\Delta r - \lambda\Delta g_a - (1-\lambda)\Delta g_b] > \Delta r$. This holds by the existence condition for PE_{ab} from Proposition 1, the fact that the coefficient of β_{ab} is positive from A2, and $\beta_{ab} > 0$. Thus $\rho_{bb} = 0$. To show that $\eta = 0$ suppose $\eta > 0$. Then conditional on an AB split, the joint payoff from A as the incumbent is at least as large as the joint payoff from B as the incumbent. This implies $\mu g_{aa} + (1-\mu)g_{ab} + p_a^\lambda + \beta_{ab}[\lambda g_{ab} + (1-\lambda)g_{bb} - p_a^\lambda] + (1-\beta_{ab})r_b \geq \mu g_{ab} + (1-\mu)g_{bb} + p_a^\lambda + \beta_{ab}[\lambda g_{aa} + (1-\lambda)g_{ab} - p_a^\lambda] + (1-\beta_{ab})r_a$. This reduces to $\lambda \leq [\mu(\Delta g_a - \Delta g_b) - (1-\beta_{ab})(\Delta r - \Delta g_b)]/\beta_{ab}(\Delta g_a - \Delta g_b)$. Substituting for μ and β_{ab} and using A1 and A2, the numerator is strictly negative while the denominator is positive. But $\lambda \geq 0$ and thus $\eta = 0$.

Consider SE. The equilibrium probability β_A of a match at the low price p_A is the ratio of type-A incumbents with vacancies to the total number of unattached workers:

$$\beta_A = m[(1-\rho_{aa})\pi_{aa} + \eta(1-\rho_{ab})\pi_{ab}] / \{n - m[(1+\rho_{aa})\pi_{aa} + (1+\rho_{ab})\pi_{ab} + (1+\rho_{bb})\pi_{bb}]\}$$

where A1 implies $0 \leq \beta_A < 1$. AA pairs receive a total payoff of $2g_{aa}$ by staying together. The joint payoff from separation is $\mu g_{aa} + (1-\mu)g_{ab} + p_A + \beta_A(g_{aa} - p_A) + (1-\beta_A)r_a$. This follows from the fact that unattached workers of type-a do not apply at the high price p_B. Staying together is strictly preferred if $(1-\mu)\Delta g_a + (1-\beta_A)[\Delta g_a + (1+\mu)\Delta g_b - \Delta r] > 0$. This holds because $\mu < 1$ and the coefficient of $1-\beta_A$ is positive by A2. Thus $\rho_{aa} = \alpha$. To show that $\eta = 0$ suppose $\eta > 0$. Then conditional on an AB split, the joint payoff from A as the incumbent is at least as large as the payoff from B as the incumbent. This implies $\mu g_{aa} + (1-\mu)g_{ab} + p_A + \beta_A(g_{ab} - p_A) + (1-\beta_A)r_b \geq g_{bb} + p_B + \beta_A(g_{aa} - p_A) + (1-\beta_A)r_a$ using: (1) type-b workers not matched at p_A get no surplus if matched at p_B; (2) type-a workers not matched at p_A do not apply at p_B; and (3) type-B incumbents announce p_B and attract only type-b applicants. The inequality reduces to $\mu\Delta g_a + (1-\mu)\Delta g_b \geq \beta_A\Delta g_a + (1-\beta_A)\Delta r$. By A2 and $\mu < 1$ the left side is less than Δg_a while the right side is at least Δg_a. This shows that $\eta = 0$. BB pairs strictly prefer separation if $2g_{bb} < g_{bb} + p_B + \beta_A(g_{ab} - p_A) + (1-\beta_A)r_b$, using the fact that type-b workers not matched at p_A get no surplus if matched at p_B. This holds iff $\beta_A > 0$, which is true iff $\pi_{aa} > 0$. Thus $\rho_{bb} = 0$ if $\pi_{aa} > 0$.

Consider PE_b. AA pairs never separate voluntarily if $2g_{aa} > g_{ab} + p_b^{\lambda} + r_a$, using the fact that unattached type-a workers never apply. This gives $2\Delta g_a + (1-\lambda)\Delta g_b > \Delta r$ which is true by A2. Thus $\rho_{aa} = \alpha$. Given separation, the joint payoff to an AB pair from having A be the incumbent is less than the payoff from having B be the incumbent if $g_{ab} + p_b^{\lambda} + r_b < g_{bb} + p_b^{\lambda} + r_a$, using the fact that type-b workers get no surplus from a match at p_b^{λ} and type-a workers never apply. This reduces to $\Delta g_b < \Delta r$ which holds by A2. Thus $\eta = 0$. BB pairs strictly prefer separation if $2g_{bb} < g_{bb} + p_b^{\lambda} + r_b$ since unattached type-b workers get no surplus. This holds iff $\lambda > 0$ which is true iff $\pi_{aa} > 0$. Thus $\rho_{bb} = 0$ if $\pi_{aa} > 0$.

Now consider part (iii). The result $\pi_{aa}' < \pi_{aa}$ follows from the transition equation using $\eta = 0$, $\rho_{aa} = \alpha < 1$, $\mu_A \leq \mu < 1$, and $\pi_{aa} > 0$. The result $\pi_{bb}' > \pi_{bb}$ for SE and PE_b follows from $\eta = 0$, $\rho_{ab} \leq \alpha < 1$, $\rho_{bb} = 0$, $\mu_B = 0$, and $\pi_{ab} > 0$. The result $\Delta W < 0$ for SE and PE_b follows from $\rho_{aa} < 1$, $\rho_{ab} < 1$, $\eta = 0$, $\mu_A < 1$, $\mu_B = 0$, $\pi_{bb} < 1$, and A2.

Finally, consider part (iv) and suppose PE_{ab} occurs at the relevant μ values. Let $X(\mu) \equiv (1-\mu)[1-\alpha + \alpha(1-\mu)^2]\Delta g_b$; $Y(\mu) \equiv \mu^2(1-\alpha)\Delta g_a$; and $Z(\mu) \equiv (1-\mu)(1-\alpha\mu^2)\Delta g_b$. Due to $\eta = 0$ AB never separates voluntarily if $2g_{ab} > \mu g_{ab} + (1-\mu)g_{bb} + p_a^{\lambda} + \beta_{ab}[\lambda g_{aa} + (1-\lambda)g_{ab} - p_a^{\lambda}] + (1-\beta_{ab})r_a$ which holds iff $(1-\mu)\Delta g_b > \lambda\Delta g_a$. Substituting the initial distribution $\pi_{aa} = \mu^2$, $\pi_{ab} = 2\mu(1-\mu)$, $\pi_{bb} = (1-\mu)^2$ into the expression for λ in the text, the inequality holds iff $X(\mu) > Y(\mu)$. Thus $\rho_{ab} = \alpha$ in this case. AB pairs are indifferent toward separation if $(1-\mu)\Delta g_b = \lambda\Delta g_a$ which implies

$$\rho_{ab} = [(1-\mu)(1-\alpha\mu^2)\Delta g_b - \mu^2(1-\alpha)\Delta g_a] / 2\mu(1-\mu)^2\Delta g_b \quad [0, \alpha].$$

AB always separates if $(1-\mu)\Delta g_b < \lambda\Delta g_a$ which holds iff $Z(\mu) < Y(\mu)$, given that the initial distribution is $\pi_{aa} = \mu^2$, $\pi_{ab} = 2\mu(1-\mu)$, $\pi_{bb} = (1-\mu)^2$. Thus $\rho_{ab} = 0$ in this case.

$X(\mu)$ and $Z(\mu)$ have $X(0) = Z(0) = \Delta g_b$ and $X(1) = Z(1) = 0$ with $X(\mu) < Z(\mu)$ for $0 < \mu < 1$. Both are decreasing on [0,1]. The function $Y(\mu)$ is increasing with $Y(0) = 0$ and

$Y(1) = (1-\alpha)\Delta g_a > 0$. This implies that there are μ_0 and μ_1 with $0 < \mu_0 < \mu_1 < 1$ such that $\rho_{ab} = \alpha$ on $(0, \mu_0)$, $0 \leq \rho_{ab} \leq \alpha$ on $[\mu_0, \mu_1]$, and $\rho_{ab} = 0$ on $(\mu_1, 1)$. It can be shown that ρ_{ab} is continuous in μ and decreasing on $[\mu_0, \mu_1]$. This yields the equilibrium locus for ρ_{ab} in Figure 2. The locus of ρ_{ab} values such that $\Delta W = 0$ is obtained from the expression for ΔW in the text. This has $\rho_{ab} = 1/2$ at $\mu = 0$, is continuous and increasing, and approaches $+\infty$ as $\mu \rightarrow 1$. The results involving μ^* in Proposition 3 follow from Figure 2.

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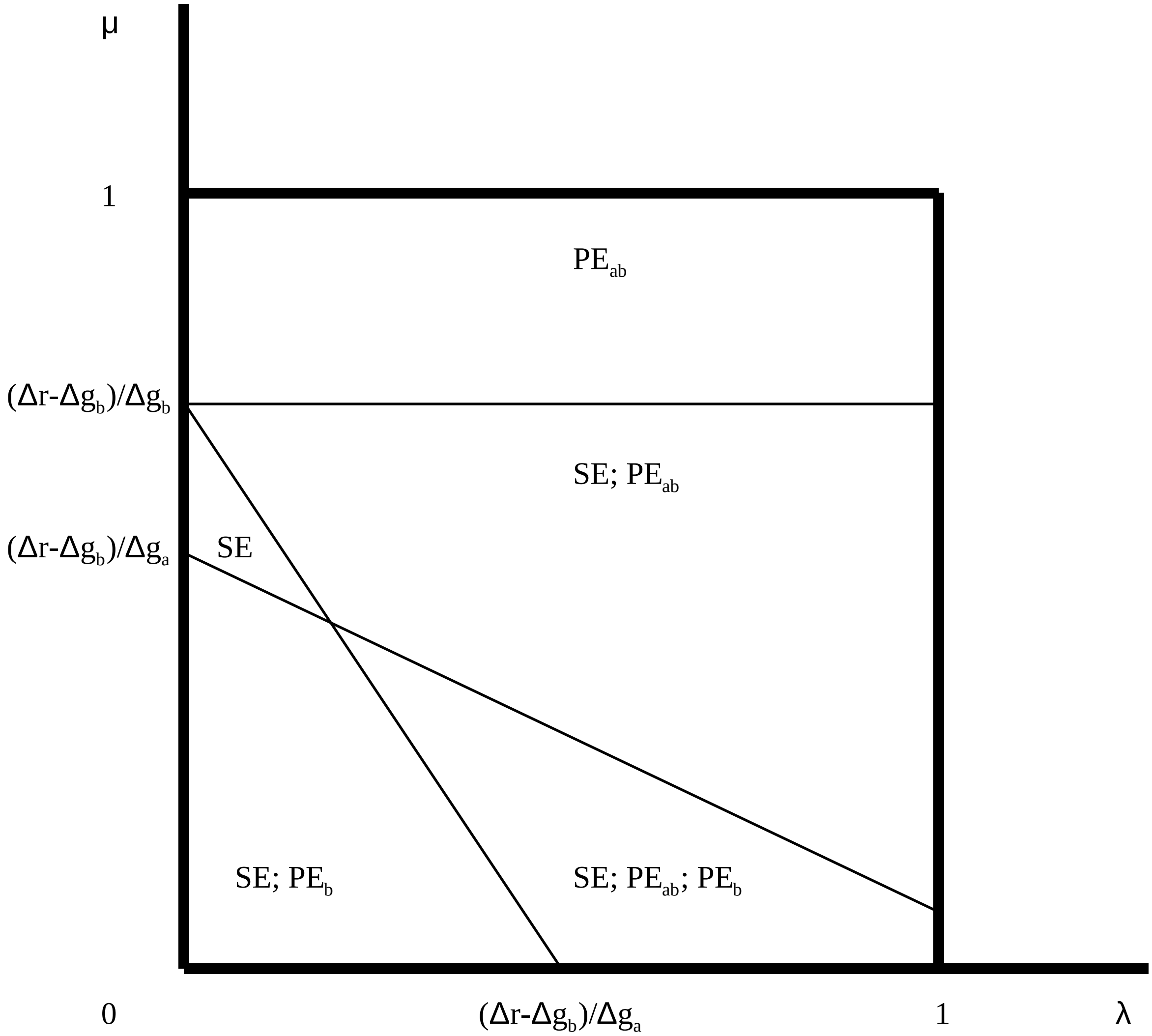

Figure 1

Existence Regions for Separating and Pooling Equilibria

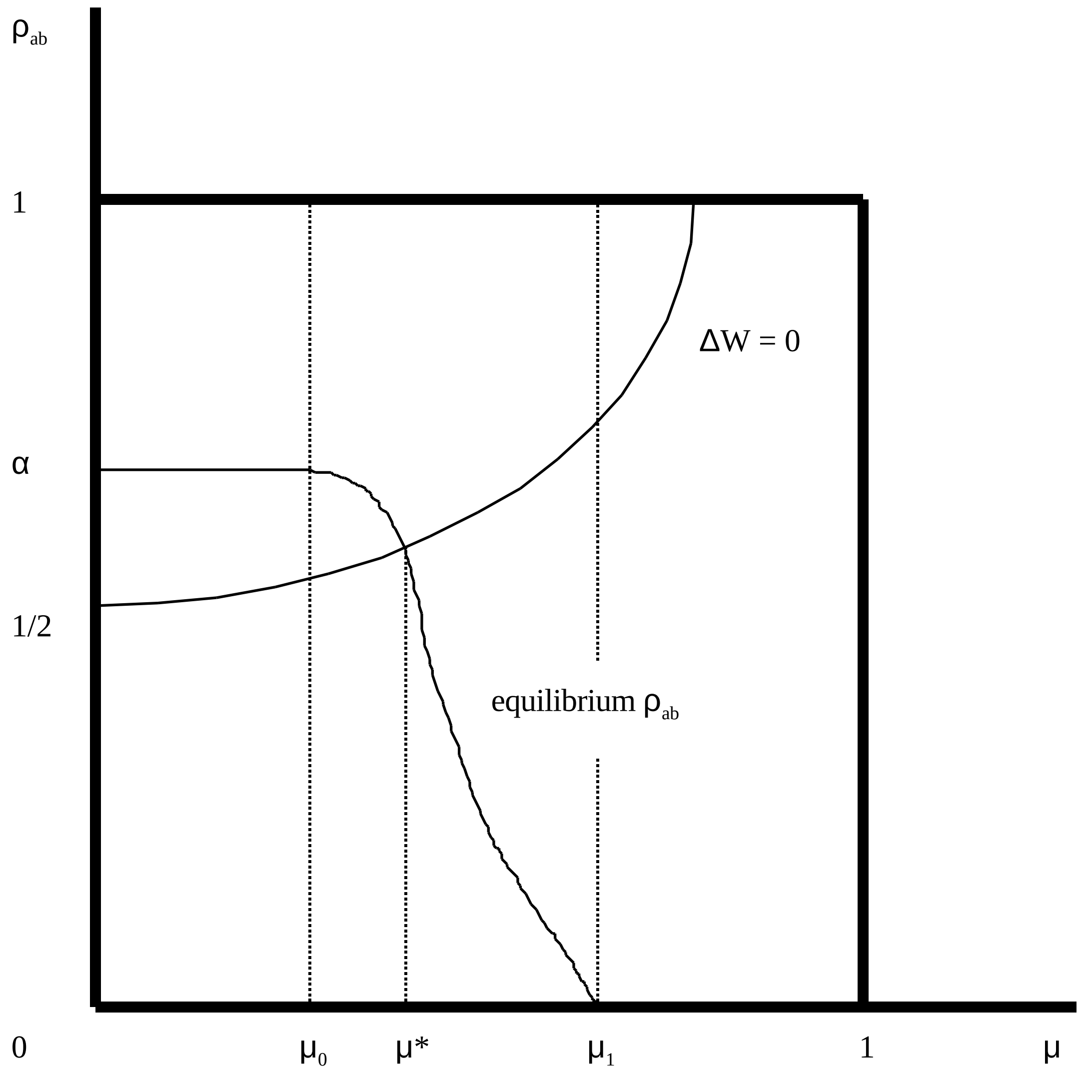

Figure 2

Welfare Analysis With Random Initial Assignment of Workers to Firms

(assuming PE_{ab} existence)