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The Effects on welfare of the Imposition of Individual Transferable Quotas on a Heterogeneous Fishing Fleet

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ABSTRACT

This paper examines the determination of effort levels, the rental price of quotas and the number of participants in a fishery managed with a total allowable catch (TAC) and individual transferable quotas (ITQs). How these variables change is determined for three phases of the management process. Potential participants in the fishery are assumed to be heterogeneous in their catching capabilities so that almost all active fishers earn positive net profits from fishing activities. It is shown that these quasirents are reduced during any of the management phases considered here. The impact of free allocations of permanent quota rights on fishers' welfare is also considered. This may increase the welfare of all fishers if the allocation is based on catch history. However, an example is provided where the welfare of some highliners is reduced when the allocation is based on effort history.

Keywords: individual transferable quotas, heterogeneous firms, quota allocation.

1. INTRODUCTION

Johnson and Libecap [7] stressed the heterogeneity of the fishers, particularly in fishing skills, in their study of the Texas shrimp fishery. They detail the practical difficulties that heterogeneity causes for fishery managers in trying to get a fishery to operate in a more efficient manner.[1] Merrifield [11] returns to this point and stresses the need to examine potential management measures for their ability to promote homogeneity among fishers. The present paper will use a theoretical bioeconomic model in the Gordon - Schaefer tradition to highlight further some aspects of the operation of a single species fishery with heterogeneous fishing firms.[2] The emphasis will be on the manner in which an open access fishery is converted to management by a total allowable catch (TAC) and individual transferable quotas (ITQs). Of particular interest will be the question of the impact on the income of fishers of these management measures.

The general economic assumptions for the Gordon - Schaefer model are as follows. Fishing firms are assumed to choose input (effort) levels to maximize current net profits from fishing. These profits are gross fishing profits (the value

[1] Other empirical studies of variations in catch rates among fishers include Hilborn and Ledbetter [6], Pálsson and Durrenberger [13] and Salvanes and Steen [14].

[2] An early theoretical discussion of variations in the opportunity cost of effort across a fishing fleet is in Copes [4].

of the catch minus the cost of the effort required to make this catch) minus the rental value of the quota required for this catch (if any). Secondly, the supply of firms to the fishery is competitive in the sense that the number of vessels active in the fishery adjusts rapidly so that the least profitable vessel makes zero economic returns from fishing. Thirdly, if fishers have to acquire quotas equal to their catch and these quotas are freely transferable in a market. then this market is competitive and adjusts rapidly to establish an equilibrium rental price for these quotas.[3]

This paper examines a model where these three conditions determine the rental price of quota v, the number N of active participants in the fishery and the optimal demands for effort k^{**} by fishing firms. The level of these variables depends on the TAC Q and the biomass M of the fish stock. The model used is a slightly more general version of a model analyzed by Terrebonne [15]. An analysis is then made of how v, N and k^{**} change during three phases of managing a fishery with a TAC and ITQs. These phases are (i) at the moment a TAC is adjusted, (ii) during a stock rebuilding period where the TAC is held constant and (iii) the long term conversion of the fishery to a socially desirable sustainable fishery. Only the third

[3] Morgan [12] discusses problems which might arise with the operation of such secondary markets.

of these phases was considered by Terrebonne. A key aspect of the model is that in any circumstance almost all active enterprises earn positive quasi-rents (net fishing profits). The impact of the above management measures on these quasi-rents will be shown to be negative. However, in the long term, social welfare as measured by the social surplus SS increases by definition. It must therefore be possible to improve the incomes of at least some enterprises by transferring to them some or all of the resource rent RR which is the difference between SS and aggregate quasi-rents.

Terrebonne discusses a transfer method which involves allocating to some group of potential participants in the fishery permanent rights to a share in the TAC for free. Then, in each period, each fishing firm must have quota rights for the period equal to its catch of fish. These rights are their initial allocation adjusted by permanent or temporary trades in the market for quotas. The firm's "full income" is its' net fishing profits plus the value of its' initial allocation of quota in the rental market for quotas. Terrebonne claims that if the initial allocation is based on the share of the catch in the preexisting fishery, then the biomass which maximizes the sustainable social surplus will also maximize each fishing firm's sustainable full income. Thus, in the long term, management of a fishery by a properly selected TAC, ITQs and the above transfer method would improve the incomes of all firms. It is shown here, however, that Terrebonne's claim is not always correct and a counterexample is provided.

In Terrebonne's analysis, both society and all fishers are assumed to have zero discount rates. The analysis in this paper covers in addition the case where society and fishers have a common but nonzero discount rate. Society is then concerned with maximizing the present value of the SS generated by the fishery and individual fishers would be interested in maximizing the present values of their own incomes from the fishery including net fishing profits and incomes generated by trades in the quota market. Assuming perfect competition in financial markets and full information about the fishery, this present value would equal the present value of their full incomes from the fishery. In both problems, there is in the long term an optimal target biomass. Terrebonne is claiming that the targets for individual fishers coincide with the social target and it is this claim that will be shown to be true only in special circumstances.

The objective of a sole owner of the fishery would be to maximize the present value of resource rents. As Terrebonne points out, this means that a sole owner with a zero discount rate would desire, in the long term, a target biomass greater than the socially optimal target biomass unlike the case of homogeneous fishers.. This result is extended here to the case where society and the sole owner have an identical but nonzero discount rate. It is also illustrated with an example.[4]

Finally, this paper examines the question of whether a free initial allocation of permanent rights to a share of the TAC improves the welfare of fishers. This is clear for the least efficient in the open access fishery if they acquire an asset with a positive value in exchange for a lifetime of near zero fishing profits. It is shown, however, that at a moment when a TAC is reduced, the more efficient fishers will have their full incomes reduced. In the longer term, if the fishery is managed to attain the socially optimal target, then all fishers will have their full incomes improved provided the allocation is based on catch history in the open access fishery. Thus, it is possible in this case that all fishers may have their welfare improved as measured by the change in the present value of their full incomes. On the other hand, if the allocation is based on effort history or is the same for all, then some of the more efficient fishers may have lower full incomes at all times than in the open access fishery and consequently will have their welfare reduced.

The intuition behind these results is that the more efficient receive initial allocations of quota which are below their desired catches in the regulated fishery. As a result, they suffer losses from having to lease quota rights as well as from

[4] Arnason [1] has proposed managing fisheries with ITQs where the TAC is set to maximize the market value of the quotas. This is equivalent to maximizing the present value of RR and so the proposal only works if the fishing fleet is homogeneous.

having to reduce their effort levels from the profit maximizing levels of the open access fishery. In the long term, these losses are offset by the cost reductions resulting from growth in the biomass. However, the losses for some from having to lease quota persist and the more an allocation method is less favourable towards the more efficient fishers, the more likely it is that these fishers will have their welfare reduced..

2. THE MODEL

The following is a review of Terrebonne's [15] model.[5] Fishing entrepreneurs are indicated by x and each has a catch function of the form $q(x) = f(k, M, x)$ when k is an index of inputs used in fishing, M is the biomass of the fish stock and the dependance of the catch on x indicates the differing fishing abilities of the entrepreneurs. The normal assumptions about the catch functions are:

$$f_k > 0, \quad f_{kk} \leq 0, \quad f_M > 0$$

Fishing profits for an entrepreneur who chooses to fish are then

[5]The model was originally developed by Clark [3] where the number of vessels was modelled as a discrete variable. Similar models may be found in Heaps and Helliwell [5] and Mattiasson [9]. In Terrebonne's version of the model the number of vessels is treated as a continuous variable for mathematical convenience. The same results can be obtained with Clark's version of the model. Boyce [2] and Matulich and Sever [11] also analyze fisheries with heterogeneous fishers. However, their fisheries are seasonal in nature and the issue of stock conservation is not addressed.

$$\pi(k, v, M, x) = (p - v)f(k, M, x) - c(k, x) \tag{1}$$

where p is the price of fish, v is the rental price of the quota needed to catch the fish, and $c(k, x)$ is the cost of using the input including the opportunity cost of the fisher. Otherwise, fishing profits are zero. It will be assumed that the fishing firms can be indexed so that $\pi_x < 0$ and $\pi_{kx} < 0$. Further, it will be assumed that $\pi(k, v, M, x)$ is strictly concave in k and that $AC = c(k, x)/f(k, M, x)$ is unbounded from above with respect to k [6] Then if the fisher's problem of choosing k to maximize profits has a nonzero solution, it has a unique solution $k^*(v, M, x)$ which satisfies the first order condition

$$\pi_k = (p - v)f_k(k^*(v, M, x), M, x) - c_k(k^*(v, M, x), x) = 0 \tag{2}$$

and moreover $\pi_{kk}(k^*) < 0$.[7]

Note that

[6]These conditions imply that $\pi < 0$ for k large so the profit maximization problem has a solution.

[7]If $\underline{k}(v, M, x)$ maximizes $A\pi = \pi(k, v, M, x)/k$ and $A\pi(\underline{k}) \geq 0$, then for $0 < k < \underline{k}$, $A\pi(k) \leq A\pi(\underline{k})$ and $\pi(k) \leq \pi(\underline{k})$. Thus $k^*(v, M, x) \geq \underline{k}(v, M, x)$.

$$\frac{\partial k^*(x)}{\partial v} = \frac{f_k}{\pi_{kk}} < 0 \quad \frac{\partial k^*(x)}{\partial M} = -\frac{(p-v)f_{kM}}{\pi_{kk}} > 0 \tag{3}$$

when $k^*(v, M, x) > 0$. In this case in addition, $\partial k^*(x)/\partial x = -\pi_{kx}/\pi_{kk} < 0$ so that effort falls with the ability of the fisher.

The fishery is otherwise assumed to operate in an open access manner.[8] The equilibrium conditions are then that given the input choices described by equation (2), the marginal fisher N should make 0 fishing profits and the catch of all fishers combined should equal the TAC Q which is assumed to be set by a management agency. That is

$$(p-v)f(k^*(v, M, N), M, N) - c(k^*(v, M, N), N) = 0 \tag{4}$$

$$\int_0^N f(k^*(v, M, x), M, x)dx = Q \tag{5}$$

These conditions determine the quota rental price and the number of active vessels in the fishery from M and Q.[9] Unlike Terrebonne [15] however, the stock is not assumed to be in biological equilibrium. The interest here is to examine the

[8] The fishery is fully open access when $v = 0$.

[9] To ensure that (4) has a solution, it should be assumed that, for all v and M, the minimum value of AC increases without bound in x so that the number of active fishers is always finite.

effects of changes in M or Q on v and N. A change in Q, M being fixed represents the adjustment to a TAC being imposed on a fishery. The intuitive comparative static results are that if Q falls, then v should increase and N should fall. An increase in M, Q being fixed represents growth in the biomass over periods where the TAC is being held constant. One would expect also that during such periods v would increase and N would fall.

To obtain these results formally, equations (4) and (5) can be differentiated implicitly taking account of the dependence of k^* on v and M. This calculation gives

$$\begin{bmatrix} -q(N) + \pi_k(N)(\partial k^*(N)/\partial v) & \pi_x(k^*(N), M, N) \\ \int_0^N f_k(\partial k^*(x)/\partial v)dx & q(N) \end{bmatrix} \begin{bmatrix} \partial v/\partial M & \partial v/\partial Q \\ \partial N/\partial M & \partial N/\partial Q \end{bmatrix}$$

$$= \begin{bmatrix} -(p-v)f_M(N) & 0 \\ -\int_0^N (f_k(\partial k^*(x)/\partial M) + f_M)dx & 1 \end{bmatrix} \qquad (6)$$

where $q(x) = f(k^*(x), M, x)$.

Using $\pi_k = 0$, the determinant of the coefficient matrix of these equations is

$$|A| = -q(N)^2 - \pi_x(k^*(N), M, N) \int_0^N f_k(\partial k^*(x)/\partial v)dx < 0 \qquad (7)$$

and using Cramer's rule the comparative static results are

$$\frac{\partial v}{\partial M} = \frac{-(p-v)f_M(N)q(N) + \pi_x(k^*(N), M, N)\int_0^N (f_k(\partial k^*(x)/\partial M) + f_M)dx}{|A|} > 0 \tag{8}$$

$$\frac{\partial N}{\partial M} = \frac{(p-v)f_M(N)\int_0^N f_k(\partial k^*(x)/\partial v)dx + q(N)\int_0^N (f_k(\partial k^*(x)/\partial M) + f_M)dx}{|A|} \tag{9}$$

$$\frac{\partial v}{\partial Q} = -\frac{\pi_x(k^*(N), M, N)}{|A|} < 0 \tag{10}$$

$$\frac{\partial N}{\partial Q} = -\frac{q(N)}{|A|} > 0 \tag{11}$$

These equations confirm three of the results mentioned above. However, it does not seem possible to get the sign of $\partial N/\partial M$ in the case of a general catch function.

Terrebonne introduced the common assumption of proportional catching technology to further his analysis of the model. To simplify the results above with respect to M, here something similar, but a little more general will be done It will be assumed that for all fishers[10]

$$f(k, M, x) = h(k, x)\phi(M) \tag{12}$$

Then using the formulas in equation (2), the numerator of $\partial N/\partial M$ is

$$(p - v)h(N)\phi' \int_0^N \frac{h_k^2 \phi^2}{\pi_{kk}} dx + h(N)\phi \int_0^N (-\frac{(p-v)h_k^2 \phi' \phi}{\pi_{kk}} + h\phi')dx$$

which simplifies to $q(N)Q\phi'/\phi > 0$. Thus in this case, the intuitive result $\partial N/\partial M < 0$ holds. A similar calculation also simplifies the other comparative static result for M and the two results can be written as

$$\frac{\partial v}{\partial M} = \frac{(p-v)\phi'(M)}{\phi(M)} + \frac{\pi_x(k^*(N), M, N)Q\phi'(M)/\phi(M)}{|A|} > 0 \tag{13}$$

$$\frac{\partial N}{\partial M} = \frac{q(N)Q\phi'(M)/\phi(M)}{|A|} < 0 \tag{14}$$

Some further effects of imposing a TAC or of holding the TAC constant for a

[10] $\phi(M)$ is assumed to strictly increasing in M and it will also be assumed that $ln\phi$ is concave in M. $\phi = M^\beta$ where $\beta > 0$ is an example.

period of time are discussed in the next section.

3. OTHER TRANSITIONAL EFFECTS

The equilibrium demand for fishing effort by entrepreneur x is determined recursively by the model in the form

$$k^{**}(M,Q,x) = k^*(v(M,Q),M,x)$$

where $k^*(v,M,x)$ is defined by equation (2). Using equation (3) and the separability assumption (12), two partial derivatives of $k^*(v,M,x)$ are:

$$\frac{\partial k^*(x)}{\partial v} = \frac{h_k \phi}{\pi_{kk}} < 0 \quad \frac{\partial k^*(x)}{\partial M} = -\frac{(p-v)h_k \phi'}{\pi_{kk}} > 0 \tag{15}$$

The effects of changes in Q or M on equilibrium fishing effort are then calculated using equations (13) and (10) as

$$\frac{\partial k^{**}(M,Q,x)}{\partial M} = \frac{\partial k^*(x)}{\partial v}\frac{\partial v}{\partial M} + \frac{\partial k^*(x)}{\partial M}$$

$$= \frac{h_k \Gamma Q \phi'(M)}{\pi_{kk}} < 0 \tag{16}$$

where $\Gamma = \pi_x(k^*(N), M, N) / |A| > 0$.

$$\frac{\partial k^{**}(M,Q,x)}{\partial Q} = \frac{\partial k^*(x)}{\partial v}\frac{\partial v}{\partial Q} = -\frac{h_k \Gamma \phi(M)}{\pi_{kk}} > 0 \tag{17}$$

Thus during a period where the TAC is fixed and the biomass is growing or at a time the TAC is adjusted downward, equilibrium fishing effort will be reduced.

At the same time, the remaining active vessels will have their profits from fishing net of the rental cost of their quotas reduced. These net profits are

$$\pi(x) = (p-v)h(k^{**}(M,Q,x),x)\phi(M) - c(k^{**}(M,Q,x),x) \tag{18}$$

Differentiating this formula gives the results

$$\frac{\partial \pi(x)}{\partial M} = \pi_k \frac{\partial k^{**}}{\partial M} - q(x)\frac{\partial v}{\partial M} + (p-v)h(x)\phi'(M)$$

$$= -q(x)\frac{\Gamma Q \phi'(M)}{\phi(M)} < 0 \tag{19}$$

(using $\pi_k = 0$ and equation (13)) and

$$\frac{\partial \pi(x)}{\partial Q} = \pi_k \frac{\partial k^{**}}{\partial Q} - q(x)\frac{\partial v}{\partial Q} = q(x)\Gamma > 0 \tag{20}$$

(using equation (10)).

These results depend, of course, on the management agency being able to collect the full market lease value of the quotas that it issues. Also of interest are the aggregate impacts on social welfare. The social surplus SS generated by the fishery is the sum of the gross fishing profits earned by fishing firms and can be written as

$$SS = \int_0^N \pi(x)dx + vQ$$

Using the equilibrium condition $\pi(N) = 0$ and equations (13) and (19)

$$\frac{\partial SS}{\partial M} = \pi(N)\frac{\partial N}{\partial M} + \int_0^N \frac{\partial \pi(x)}{\partial M}dx + \frac{\partial v}{\partial M}Q$$

$$= (p - v)Q\phi'(M)/\phi(M) > 0 \tag{21}$$

Similarly, using equations (10) and (20)

$$\frac{\partial SS}{\partial Q} = \pi(N)\frac{\partial N}{\partial Q} + \int_0^N \frac{\partial \pi(x)}{\partial Q}dx + \frac{\partial v}{\partial Q}Q + v = v \geq 0 \tag{22}$$

Thus, during a period where the TAC is held constant and M is growing, aggregate social welfare will improve. However, if a TAC is first imposed or is at some time reduced the opposite will occur. One implication is that even if the quotas are given out free so that fishers get the whole of the social surplus, at least some of the fishers must be made worse off by the imposition of a TAC. This situation can be further explored by defining what Terrebonne calls full profits to be the income fishers would receive if they were given for free a permanent quota which is a share $s_0(x)$ of the TAC and which they retain in perpetuity. This full income is the sum of gross profits from fishing and the gains or losses from trades in the rental market for quotas. That is, for those fishers who remain active in the fishery

$$\pi^*(x) = ph(k^{**}(M,Q,x),x)\phi(M) - c(k^{**}(M,Q,x),x) + (s_0(x) - s(x))vQ \quad (23)$$

where $s(x) = q(x)/Q$ is the share of the fisher in the aggregate catch after trades are made in the quota market. At the moment a TAC is reduced , all fishers will lower their fishing effort (equation (17)). This will lower their gross profits from fishing as by equation (2), marginal gross profits with respect to effort are positive. In addition, some fishers must be leasing quota in excess of their permanent allocation. This causes them an additional loss in their full income.

The formal transitional effects on full income are as follows. Note that

$$\pi^*(x) = \pi(x) + s_0(x)vQ$$

Then by equations (19), (13), (20) and (10)

$$\frac{\partial \pi^*(x)}{\partial M} = \frac{\partial \pi(x)}{\partial M} + s_0(x)\frac{\partial v}{\partial M}Q$$

$$= (s_0(x)(p - v) + (s_0(x) - s(x))\Gamma Q)Q\frac{\phi'(M)}{\phi(M)} \tag{24}$$

$$\frac{\partial \pi^*(x)}{\partial Q} = \frac{\partial \pi(x)}{\partial Q} + s_0(x)(\frac{\partial v}{\partial Q}Q + v)$$

$$= s_0(x)v + (s(x) - s_0(x))\Gamma Q \tag{25}$$

The effect on full profits depends on how $s(x)$ compares to the share of the permanent allocation of quota. The imposition of a TAC will initially reduce the full income of those who have $\partial \pi^*(x)/\partial Q > 0$. This group includes those who wish to increase their share of the aggregate catch (who have $s(x) > s_0(x)$) and there must be some such fishers if the number of active fishers is reduced. Figure 1 shows, however, there are cases where some still active fishers have their shares of the aggregate catch reduced by the imposition of a

FIG. 1. here

TAC.[11] This figure and all subsequent figures are based on the example presented in Appendix A.[12] Figure 2 shows the change in full profits which occurs in case 2 of Figure 1 if $s_0(x)$ is based on catch history. The remainiing active fishers are those whose changes are to the left of the kinks in the curves. Immediately after the imposition of the TAC, all these fishers have their full profits reduced as their shares in the catch have increased.[13] Those who choose to lease out or sell their entire allocation will get its rental market value as their full income and their changes in full incomes are shown to the right of the kinks in the curves. The least efficient in the open access fishery will definitely gain from the imposition of a TAC as their open access profits are near zero. The figure shows however that some of the exiters may have their full profits reduced. However, subsequent growth in the biomass while the TAC is held fixed, which causes further exit from the fishery, improves the full profitability of all. In case 1, not shown, all fishers are better off than in open access after the 25% increase in the biomass. The question of what happens to full profits in the longer term will be addressed below.

[11] All figures in this paper were drawn using the mathematical software Maple7. The Maple text files used, stgr.txt and ltgr.txt can be downloaded from the URL http://www.sfu.ca/~heaps/hetfish.

[12] The example contains 3 parameters: t the open access biomass, m an index of heterogenity and w an economic parameter. In both cases $t = 2/5$ and $m = 10$. In case 1, $w = 1/2$ and in case 2, $w = 2$. The TAC is set at 75% of the open access catch.

[13] In case 1, with the TAC set at 90% of the open access catch, some of the less efficient but still active fishers have their full profits increased.

FIG. 2. here

4. LONG TERM EFFECTS

The purpose of this section is to compare the open access fishery with the bioeconomic equilibrium in the fishery where the TAC is set at its socially optimal target level. This can be accomplished by determining how the biologically sustainable equilibria given by the additional condition $Q = g(M)$ change when M changes. Here $g(M)$ is the natural growth function of the fish stock and $Z^s(M) = Z(M.g(M))$ will be used to denote the level of a sustainable variable.[14] The change in Z^s with respect to M can be computed as $\partial Z^s/\partial M = \partial Z/\partial M + g'(M)\partial Z/\partial Q$ where the derivatives on the right hand side are those computed in the previous two sections. Table I gives the results of these calculations. The signs of the results depend on the sign of $D = g(M)\phi'(M)/\phi(M) - g'(M)$. It will be assumed that this expression is positive for all M not less than the open access biomass.[15] Then the long term effects of an increase in the sustainable biomass on v^s, N^s, $k^{**s}(x)$ and $\pi^s(x)$ are qualitatively the same as the transitional effects of M increasing

[14] $g(M)$ is assumed to be strictly concave in M.

[15] This assumption definitely holds for an interval of M beginning below the biomass which maximizes $g(M)$. Another justification, given in Terrebonne [15], is that open access equilibrium should be dynamically stable. Here suppose the configuration of the fishing fleet is fixed. The dynamics of the biomass would be given by $\dot{M} = g(M) - \phi(M)R$ where R is fixed. At the equilibrium given by $g(M)/\phi(M) = R$, $g'(M) - \phi'(M)R = -D < 0$ is required for local stability.

with Q fixed and the same comments apply. The condition which determines the

Table I

The Sustainable Derivatives

Z^s	$\partial Z^s/\partial M$
v^s	$(p - v^s)\phi'/\phi + \Gamma D > 0$
N^s	$q^s(N)D/\left\|A\right\| < 0$
$k^{**s}(x)$	$h_k \Gamma D \phi / \pi^s_{kk} < 0$
$\pi^s(x)$	$-q^s(x)\Gamma D \leq 0$
SS^s	$(p - v^s)g\phi'/\phi + v^s g'$
RR^s	$\partial SS^s/\partial M + g\Gamma D$
$\pi^{*s}(x)$	$s_0(x)\partial SS^s/\partial M + (s_0(x) - s^s(x))g\Gamma D$

$$\Gamma = \pi^s_x(k^{**s}(N), M, N^s)/\left|A\right| > 0 \;:\; D = g\phi'/\phi - g' > 0$$

long term socially optimal target biomass M^* is $\partial SS^s/\partial M = \delta v^s$ where δ is the social discount rate. This condition can be interpreted as the marginal sustainable benefit (ΔSS^s) to society of increasing M by reducing the TAC should equal the social rate of return on the financial capital (v^s) that the reduction in the TAC would otherwise have created. In contrast, a sole owner of the fishery would seek to maximize the present value of the sustainable resource rent generated by the fishery which is the rental price of quotas times the TAC, that is $RR^s = v^s g(M)$. From Table I,, $\partial RR^s/\partial M > \partial SS^s/\partial M$ so when $M = M^*$, $\partial RR^s/\partial M > \delta v^s$ and the present value of resource rent (discount rate $= \delta$) is maximized in the long term by a target biomass M_r which is higher than the socially optimal target biomass as was shown in the case $\delta = 0$ by Terrebonne[15]. Figure 3 provides an illustration of the difference between these 2 biomasses in this case.[16]

A major problem in establishing an individual transferable quota system for a fishery that was previously open access is deciding how to supply quota to the fishing entrepreneurs. As illustrated in the previous section, the allocation method determines how the benefits of rationalizing the fishery are distributed among fishers and the regulators. The ITQs could be sold to fishers in each period at a price which balances the fleet demand for quotas with the TAC. However, such a

[16]The parameter values are $t = 2/5$, $m = 10$, $\delta = 0$ and w is varied from 0 to 50.

FIG. 3. here

scheme would be very unpopular because as shown above it reduces the net profits $\pi(x)$ of all the fishers. Kaufman and Geen [8] state that in practise permanent rights to quotas are allocated at zero or very low cost according to some administrative formula. One proposal would be to assign permanent quotas to fishers equal to their socially optimal shares in the TAC. This would mean that some fishers would have to be given a zero allocation in order to reduce vessel numbers while the other fishers would have to be given a different share of the total catch than they had historically. Such an allocation rule would also be unlikely to be politically acceptable.

The allocation formulas discussed by Kaufman and Geen are based on some combination of historical catches and the historical use of some inputs. Given such a formula, an issue raised in Terrebonne [15] is what long term biomass would be preferred by individual fishing enterprises. Their objective should be the maximization of the present value of their full incomes $\pi^{*s}(x)$ where it will be assumed that they use the same discount rate as is used by society. He claims that if the allocation formula is based solely on catch history, then all enterprises will prefer a target biomass equal to the socially optimal target biomass. This is not, however, correct in general. The individual enterprise receives $\pi^{*s}(x)$ as its share of the social surplus and $s_0(x)v^s$ as its share of the financial capital created by catching fish. Its preferred long term target biomass $M_{fp}(x)$ is thus given by $\partial\pi^{*s}(x)/\partial M = \delta s_0(x)v^s$ when $s_0(x) > 0$.

One group of quota receivers would rent out (or sell) their entire allocation and receive a full income that was proportional to the market value of the aggregate quota $v^s g(M)$. These entrepreneurs would be in favour of reducing the TAC below its socially optimal target level to $g(M_r)$.

The effect of increasing M on sustainable full income $\pi^{*s}(x)$ is shown in Table I. When $M = M^*$ the socially optimal target biomass, $\partial\pi^*(x)/\partial M = \delta s_0(x)v^s + (s_0(x) - s^*(x))g\Gamma D$ where $s^*(x)$ is the share of the enterprise in the long term socially optimal TAC. Thus if the allocation is based on the socially optimal target shares, the second term is zero and the present value of $\pi^{*s}(x)$ is indeed maximized in the long term at $M = M^*$ for those who receive a nonzero allocation However, those who participated in the open access fishery and are now excluded would be made worse off. If the allocation is based instead on catch history in the open access fishery, consider those who wish to remain active in the fishery when $M = M^*$. It must be the case that $s_0(x) < s^*(x)$ for at least some of these surviving entrepreneurs and such a fisher would prefer a long term target stock size $M_{fp}(x)$ that was smaller than M^* since $\partial\pi^{*s}(x)/\partial M < \delta s_0(x)v^s$ at $M = M^*$.

Figure 4 shows 2 examples of the target biomasses associated with maximizing

FIG. 4. here.

the present value of full profits when the permanent allocation share $s_{q0}(x)$ is based on catch history.[17] These biomasses are different for different fishers. They are also below or above M^* when the socially optimal target share is greater than or less than the initial allocation.[18]

5. WHO GAINS?

In a fishery where permanent rights to quotas are given out for free, the original fishers receive the entire increase in the present value of the social surplus. At least some of them will thus have their welfare as measured by the present value of their full incomes improved. Indeed, if all the original fishers receive an allocation, then those most marginal in the open access situation go from nearly zero returns to the present value of a positive share of the resource rent and thus clearly have their welfare improved. However, the same cannot be so easily concluded foe

[17] Here $\delta = 0$. In addition, the cases for this and the next two figures are the same as the cases for Figure 1. A plot of the changes in the shares from open access to the socially optimal target fishery for these two cases is similar to Figure 1.

[18] For a Cobb-Douglas production function $f = h(x)k^{\alpha}M^{\beta}$, $s_0(x)/s(x)$ does not depend on x. All remaning active fishers then do agree on which stock size is most beneficial for them but it is below M^*. For details, see http://www.sfu.ca/~heaps/hetfish/cdex.pdf.

those highliners who initially have their full incomes reduced by the imposition of a TAC. A full analysis of what happens to the welfare of individual fishers under a management plan for converting the fishery from open access conditions to socially optimal conditions is beyond the scope of this paper as it depends on the details of the plan and on the dynamics of stock rebuilding. However, one can compare open access full incomes with socially optimal full incomes to see if there are latter gains in income which may offset the early losses of some of the highliners. In a seasonal model of a fishery, Matulich and Sever [11] show that there are initial allocations of ITQs that increase the full incomes of all. This turns out also to be the case in a fishery where a permanent allocation is based on catch history. A proof is provided in Appendix B. Figure 5 shows two examples of the gains in full incomes received by fishers from moving from open access to free ITQs. It is interesting to see that in one of the cases marginal fishers actually benefit more than do some of the highliners.

The case where the initial allocation of shares $s_{k0}(x)$ is based on the fisher's application of effort in the open access fishery will also be examined. The ratio of the two allocations (s_q/s_k) is a constant times $h(k^{**}(x), x)/k^{**}(x)$ which is catch per unit effort by fisher x in the open access fishery. An increase in x will directly reduce this CPUE but will also reduce k^{**} which will cause $h(k)/k$ to rise.

A comparison of the two allocations therefore seems to require some additional assumptions about the technology. The following assumptions are satisfied by the examples mentioned above. First h is separable in k and x. That is $h = d(k)e(x)$

FIG. 5. here.

where d is strictly concave and $e'(x) < 0$ for all x. Secondly $kd'(k)/d(k)$ is nonincreasing in k and thirdly $c(k, x)$ is independent of x and concave in k.[19] Then, $h(k^{**})/k^{**} = d(k^{**})e(x)/k^{**}$. From equation (2), $pd'(k^{**})e(x)/c'(k^{**})$ depends only on the open access biomass. Using this fact to eliminate $e(x)$, the ratio $s_{q0}(x)/s_{k0}(x)$ is a constant times

$$\frac{d(k^{**})}{k^{**}d'(k^{**})}c'(k^{**})$$

The two terms in this expression are nondecreasing in k^{**} by the assumptions made and since k^{**} is lower for higher x this means $s_{q0}(x)/s_{k0}(x)$ is decreasing in x. Thus the effort allocation favours lowliners over highliners as compared to the catch allocation. Figure 6 illustrates this comparison for case 2 of the example. In this particular case, all fishers have higher full incomes in the long term than

[19]For a concave $d(k)$, both the average and marginal functions of d are decreasing in k. The assumption says that the average falls no more quickly than the marginal in percentage terms.

they did in the full open access equilibrium. However, there are other cases in which highliners actually have lower full incomes when the free initial allocation of shares is based on effort.[20]

A third allocation formula which is even more favourable towards the lowliners

FIG. 6. here.

is to give all the participants in the original fishery the same share of the TAC. This formula may also reduce the long term full incomes of the highliners. The interesting observation here is that there are cases of the example where the long term full incomes of all entrepreneurs are improved by the application of this formula.[21]

6. CONCLUSIONS

The immediate effects of the imposition of a TAC on a heterogeneous fishery include a nonzero price in the quota market and a reduction in the social surplus. This means that even if fishers are given free quotas, some of the original fishers will initially have lower full incomes. Other effects include falling effort levels, falling net profits from fishing and falling numbers of active vessels. Over a period

[20]For example, $t = 2/5$, $m = 25$ and $w = 0$.
[21]For example, $t = 2/5$, $m = 1$ and $w = 2$.

during which the TAC is held constant and the biomass is growing, the effects are the same except that the social surplus will grow.

Another significant point is that before and after the introduction of ITQs, different firms have different shares of the aggregate catch. These shares tend to increase with the change in management because of the reduction in the number of active firms but this effect is offset by the reduced effort levels. An example was given where the shares of some of the less efficient firms were actually reduced. Full profits are net fishing profits plus the rental market value of the initial allocation of quota to the firm, that is the income the firm would receive if there was a free permanent allocation of quota shares which was retained in perpetuity. The transitional effects on full profits were shown to depend on how the share in the initial allocation compares to the share in the aggregate catch. This reinforced the point that even with free quota shares, some fishers will initially have lower full incomes.

The long term effects also included reductions in fishing efforts, net fishing profits and vessel numbers. The issue of whether a free allocation of quota shares will improve the welfare of all fishers as measured by the present value of their full incomes was addressed. It was shown that if the TAC is set at its socially optimal target level, then this may happen if the initial allocation is based on catch history but may not happen if this allocation is based on effort history or is the same for all. Finally, Terrebonne's [15] claim that the socially optimal target biomass will also maximize, in the long term, the welfare of all individual entrepreneurs was shown to be true only in special circumstances.

APPENDIX A

The example used to illustrate some of the points made in this paper uses the production function

$$f = \ln(1 + ak)M/(1 + mx) \tag{A1}$$

which satisfies the assumptions laid out for a catch function in Section 2. That is $f(0, M, x) = 0$, $f_k = aM/(1 + mx)/(1 + ak) > 0$, $f_{kk} = -a^2M/(1 + mx)/(1 + ak)^2 < 0$, $f_M = \ln(1 + ak)/(1 + mx) > 0$, $f_x = -\ln(1 + ak)M/(1 + mx)^2 < 0$ and $f_{kx} = -aM/(1 + mx)^2/(1 + ak) < 0$.

The profit function is $\pi(v, k, M, x) = (p - v)\ln(1 + ak)M/(1 + mx) - rk - y$. By (2). the optimal choice of k (when > 0) is then given by

$$1 + ak^*(x) = \frac{a(p - v)M}{r(1 + mx)} \tag{A2}$$

The output of and profits earned by the x^{th} fisher with this choice of the input are

$$q(x) = \frac{M}{1 + mx}\ln(\frac{a(p - v)M}{r(1 + mx)}) \tag{A3}$$

$$\pi(x) = \frac{(p - v)M}{1 + mx}\ln(\frac{a(p - v)M}{er(1 + mx)}) + r/a - y \tag{A4}$$

Putting $\pi(N) = 0$, this condition for open access equilibrium can then be written as

$$(p-v)M/(1+mN) = \frac{r}{a}e^w \tag{A5}$$

where w is the unique nonnegative solution of the equation $(w-1)e^w = (ya)/r - 1$ and has values ranging from 0 to ∞. Using this condition. the equilibrium demand for effort is $1 + ak^{**}(x) = e^w(1+mN)/(1+mx)$ and output is $q(x) = M(w + \ln((1+mN)/(1+mx)))/(1+mx)$. In addition fishing costs are

$$C(x) = rk^{**}(x) + y = \frac{r}{a}e^w(w - 1 + (1+mN)/(1+mx)) \tag{A6}$$

and fishing net revenues are

$$R^n(x) = (p-v)q(x) = \frac{r}{a}e^w\frac{1+mN}{1+mx}(w + \ln((1+mN)/(1+mx))) \tag{A7}$$

The second equilibrium condition $\int_0^N q(x)dx = Q$ can now be shown to have the explicit solution

$$\ln(1+mN) = \sqrt{w^2 + 2mQ/M} - w \tag{A8}$$

Note that as w increases. N decreases as the fishery becomes less profitable.

The full open access equilibrium will be denoted by t and for the purposes of illustration will be treated as a basic parameter together with m and w. From the first equilibrium condition (A5) ($v = 0$) and assuming that the natural growth function of the biomass is $g(M) = M(1 - M)$,

$$pt = \frac{r}{a}e^{w}(1 + mN_t) \tag{A9}$$

where N_t is the number of vessels in the open access fishery and $\ln(1+mN_t) = \sqrt{w^2 + 2m(1-t)} - w$. For the fishery to be profitable at all, it must be the case that $t < 1$. As $\exp(\sqrt{w^2 + 2m(1-t)})/t$ is decreasing in t, this requires that $pa/r > e^w$.

The equilibrium condition $\pi(N) = 0$ can now be written as

$$p - v = \frac{pt(1 + mN)}{M(1 + mN_t)} \tag{A10}$$

and from (A6) and (A7) the net fishing profits of vessel x are

$$\pi(x) = \frac{pt}{(1 + mx)(1 + mN_t)}((w - 1)m(N - x) + (1 + mN)\ln(\frac{1 + mN}{1 + mx})) \tag{A11}$$

The social surplus can then be calculated as fleet gross revenue pQ minus

$\int_0^N C(x)dx$. The result of this calculation, making use of (A9) is

$$SS = pQ - \frac{pt}{m(1+mN_t)}((w-1)mN + (1+mN)\ln(1+mN)) \qquad \text{(A12)}$$

Now from (A8), the sustainable number of active vessels is given by $\ln(1 + mN^s) = \sqrt{w^2 + 2m(1-M)} - w$ and thus $\partial N^s/\partial M = -(1+mN^s)/(w + \ln(1 + mN^s))$. Using this formula and (A12) with $Q = M(1-M)$ gives

$$p^{-1}\partial SS^s/\partial M = 1 - 2M) + t(1+mN^s)/(1+mN_t)$$

Equating this with $p^{-1}\delta v^s = 1 - t(1+mN^s)/(1+mN_t)/M$ (by (A10)) yields an equation whose solution is the socially optimal long term target biomass. It can be rearranged as[22]

$$\frac{(2M+\delta-1)M}{M+\delta} = t\frac{! + mN^s}{1+mN_t} \qquad \text{(A13)}$$

From (A10) the sustainable rents earned in the fishery are

$$RR^s = v^s M(1-M) = p(M - t\frac{1+mN^s}{1+mN_t})\,(1-M)$$

[22]It can be shown that this equation has a unique solution in the interval (a, b) where $a = \max\{(1-\delta)/2, t\}$ and $4b = t + 1 - \delta + \sqrt{(t+1-\delta)^2 + 8t\delta}$ when $0 < t < 1$.

Differentiating this expression yields

$$p^{-1}\partial RR^s/\partial M = 1 - 2M + \frac{t(1+mN^s)}{(1+mN_t)}(1 + \frac{m(1-M)}{w + \ln(1+mN^s)})$$

so that the target biomass for maximizing the present value of RR^s, given by $\partial RR^s/\partial M = \delta v^s$, solves

$$\frac{(2M+\delta-1)M}{M+\delta} = t\frac{1+mN^s}{1+mN_t}(1 + \frac{mM(1-M)}{(M+\delta)(w+\ln(1+mN^s))}) \qquad \text{(A14)}$$

and which is greater than M^* provided $0 < t < 1$, that is that positive returns can be obtained from the fishery. A similar procedure applied to the formula (A11) for $\pi(x)$ gives

$$p^{-1}\partial\pi(x)^s/\partial M = -\frac{mt}{1+mx}\frac{1+mN^s}{1+mN_t}\frac{w+\ln((1+mN^s)/(1+mx)}{w+\ln(1+mN^s)}$$

From the formula for $q(x)$ above, the shares of the fishers in the total catch of the fishery are

$$s(x) = \frac{q(x)}{Q} = \frac{M}{Q(1+mx)}(\ln(\frac{1+mN}{1+mx}) + w) \qquad \text{(A15)}$$

where $Q = M(1-M)$ in the long term. Thus

$$p^{-1}\partial\pi(x)^s/\partial M = -t\frac{1+mN^s}{1+mN_t}\frac{m(1-M)s(x)}{w+\ln(1+mN^s)}$$

The formula determining the optimal target biomass for maximizing the present value of full profits of fisher x is

$$\partial\pi^*(x)^s/\partial M = \partial\pi(x)^s/\partial M + s_0(x)\partial RR^s/\partial M = \delta s_0(x)v^s$$

Using the formulas derived above this can be reorganized to

$$\frac{(2M+\delta-1)M}{M+\delta} = t\frac{1+mN}{1+mN_t}(1+\frac{mM(1-M)}{(M+\delta)(w+\ln(1+mN))}(1-\frac{s(x)}{s_0(x)}) \quad \text{(A16)}$$

The initial allocation of shares in the TAC based on historical catches is

$$s_{q0}(x) = \frac{1}{(1-t)(1+mx)}(\ln(\frac{1+mN_t}{1+mx})+w) \quad \text{(A17)}$$

and using the formula for $k^{**}(x)$ above the shares based on historical input use are

$$s_{k0}(x) = \frac{m(e^w(1+mN_t)/(1+mx)-1)}{e^w(1+mN_t)\ln(1+mN_t)-mN_t} \quad \text{(A18)}$$

APPENDIX B

Proof that full incomes increase in the long term. Suppose some fishing firm has a lower full income when the biomass is in biological equilibrium at M^* than it had in the open access fishery. Let v^* be the sustainable rental price of quota when $M = M^*$. Then

$$(p - v^*)h(k^{**})\phi(M^*) - c(k^{**}) + s_o(x)v^*g(M^*) < ph(k_t^{**})\phi(t) - c(k_t^{**})$$

where k^{**}, k_t^{**} maximize profits when $M = M^*$ and $M = t$ the open access level of the biomass respectively. It is also then the case that

$$(p - v^*)h(k_t^{**})\phi(M^*) - c(k_t^{**}) \leq (p - v^*)h(k^{**})\phi(M^*) - c(k^{**})$$

Combining the two inequalities leads to

$$(p - v^*)h(k_t^{**})\phi(M^*) + s_o(x)v^*g(M^*) < ph(k_t^{**})\phi(t)$$

Now if the initial allocation is based on historical catch shares, $s_{q0}(x) =$

$h(k_t^{**})\phi(t)/g(t)$ and it must be the case that

$$(p - v^*)g(t)\phi(M^*)/\phi(t)' + v^* g(M^*) < pg(t) \tag{B1}$$

Since g/ϕ is decreasing in M by the assumption that $D > 0$ (see Table I), this implies that $g(M^*) < g(t)$.

Now the socially optimal biomass satisfies the condition

$$(p - v^*)g(M^*)\phi'(M^*)/\phi(M^*) + v^* g'(M^*) = \delta v^*$$

Thus

$$v^* = \frac{pg(M^*)\phi'(M^*)}{g(M^*)\phi'(M^*) + (\delta - g'(M^*))\phi(M^*)}$$

$$p - v^* = \frac{p(\delta - g'(M^*))\phi(M^*)}{g(M^*)\phi'(M^*) + (\delta - g'(M^*))\phi(M^*)}$$

Substituting these expressions in the inequality (B1) above gives

$$(\delta - g'(M^*))g(t)\phi(M^*)^2/\phi(t)' + g(M^*)^2\phi'(M^*) < g(t)(g(M^*)\phi'(M^*) + (\delta - g'(M^*))\phi(M^*))$$

This inequality can be reorganized to

$$g(t)(\delta - g'(M^*))((\phi(M^*)/\phi(t\acute{)}) - 1) < g(M^*)(g(t) - g(M^*))\phi'(M^*)/\phi(M^*) \quad \text{(B2)}$$

Now if $g(M)$ is concave

$$\frac{g(t) - g(M^*)}{M^* - t} \leq -g'(M^*) \leq \delta - g'(M^*) \quad \text{(B3)}$$

Furthermore if $\ln \phi(M)$ is concave

$$\frac{\phi'(M^*)}{\phi(M^*)}(M^* - t) \leq \ln \phi(M^*) - \ln \phi(t)$$

Then applying the inequality $\ln x \leq x - 1$ if $x \geq 1$ to $x = \phi(M^*)/\phi(t)$

$$\frac{\phi'(M^*)}{\phi(M^*)}(M^* - t) \leq \frac{\phi(M^*)}{\phi(t)} - 1 \quad \text{(B4)}$$

Multiplying inequality (B3) with inequality (B4) and then further multiplying by the inequality $g(M^*) < g(t)$, which was derived from (B1), contradicts the critical inequality (B2) above.

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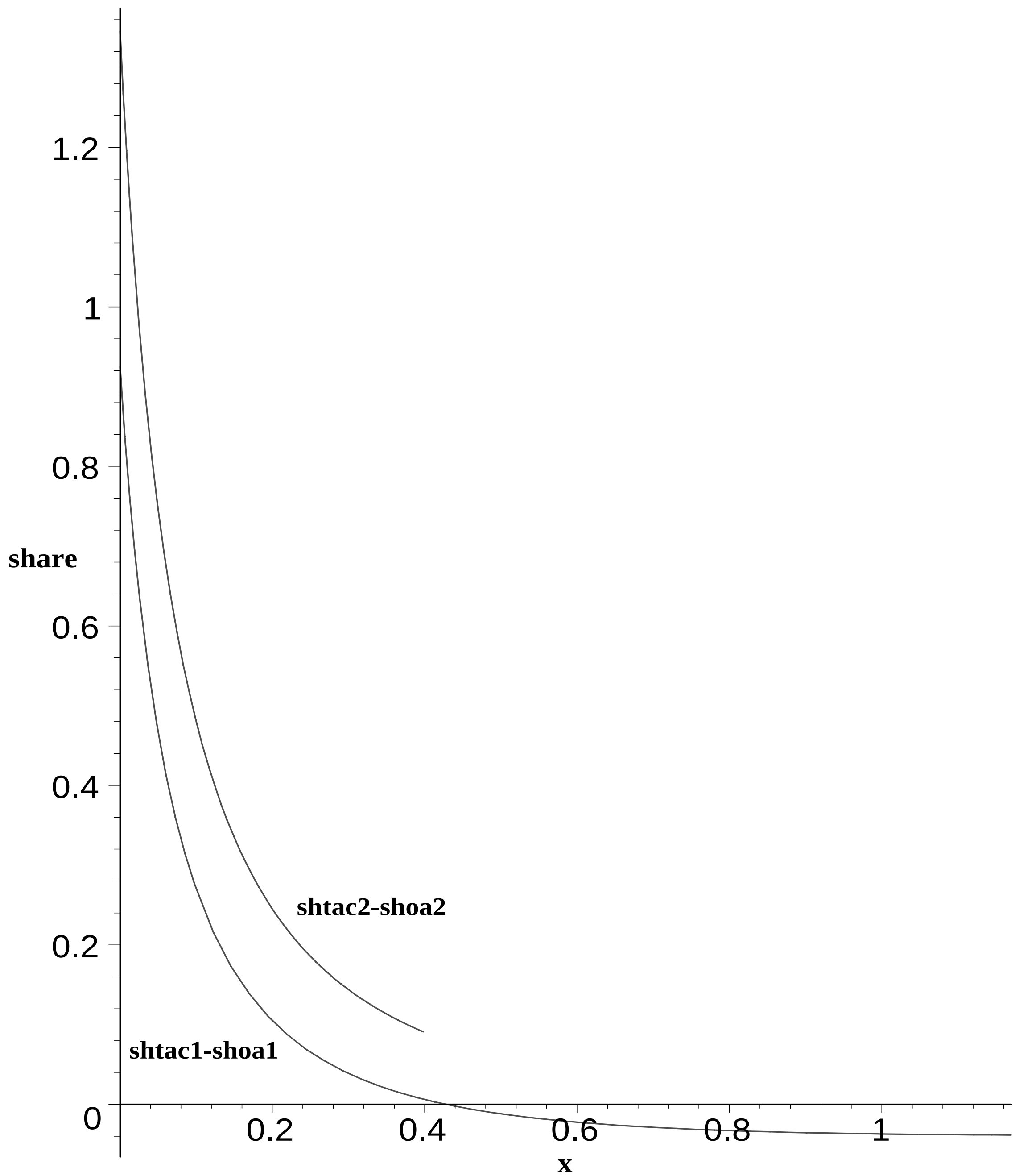

FIG. 1. Change in shares.

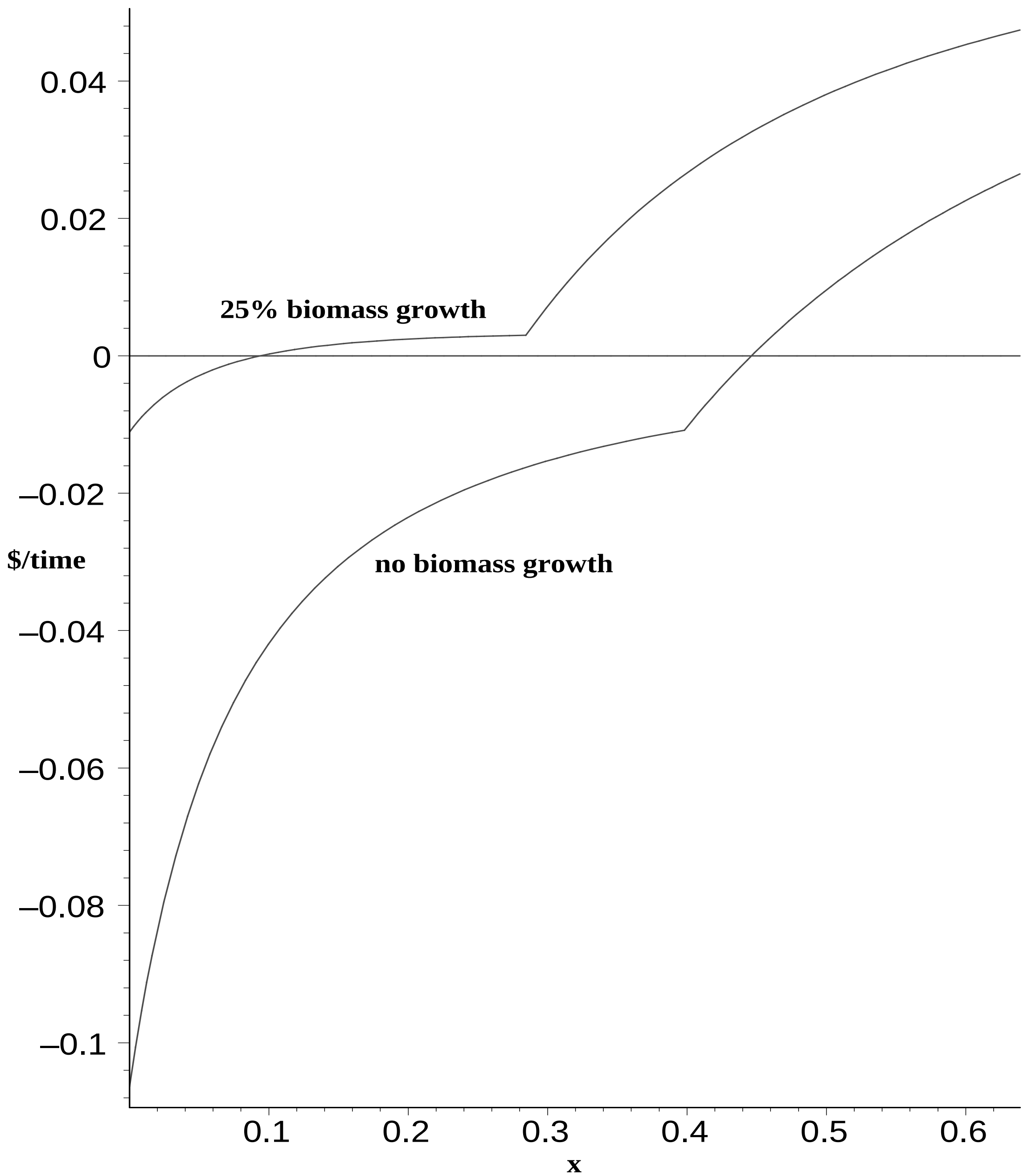

FIG. 2. Change in full profits.

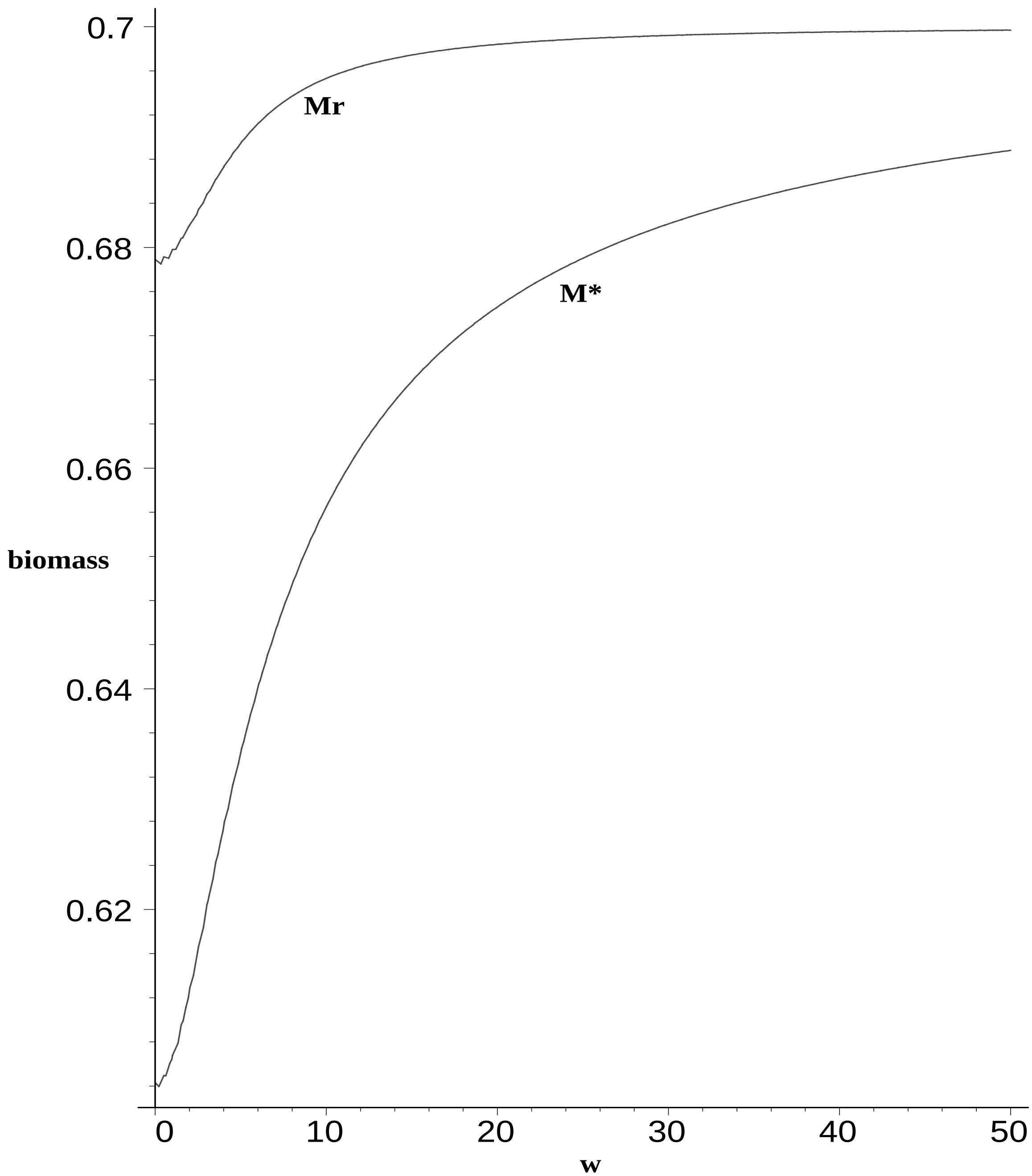

FIG. 3. Social surplus and rent maximizing biomasses.

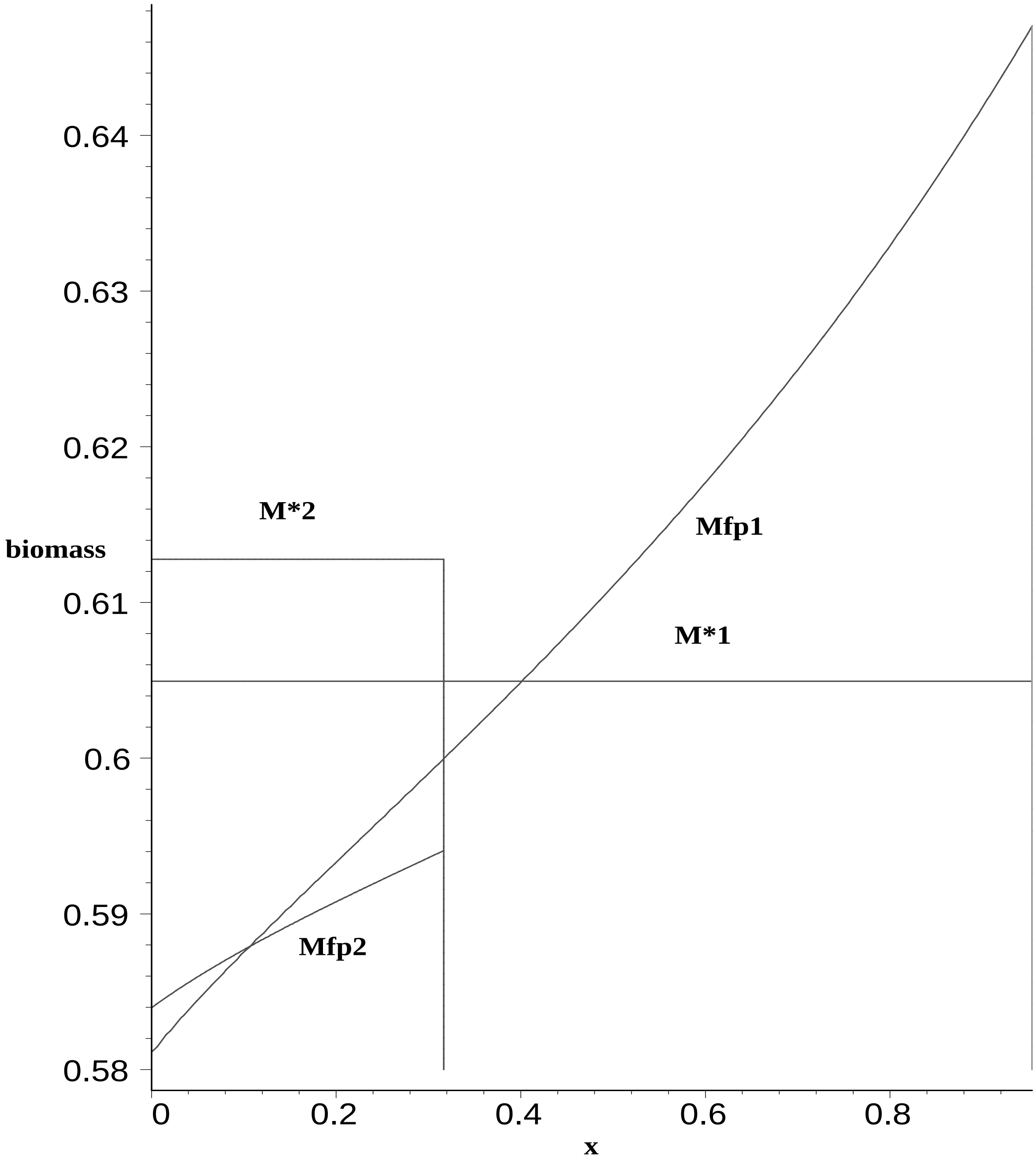

FIG. 4. Optimal biomasses.

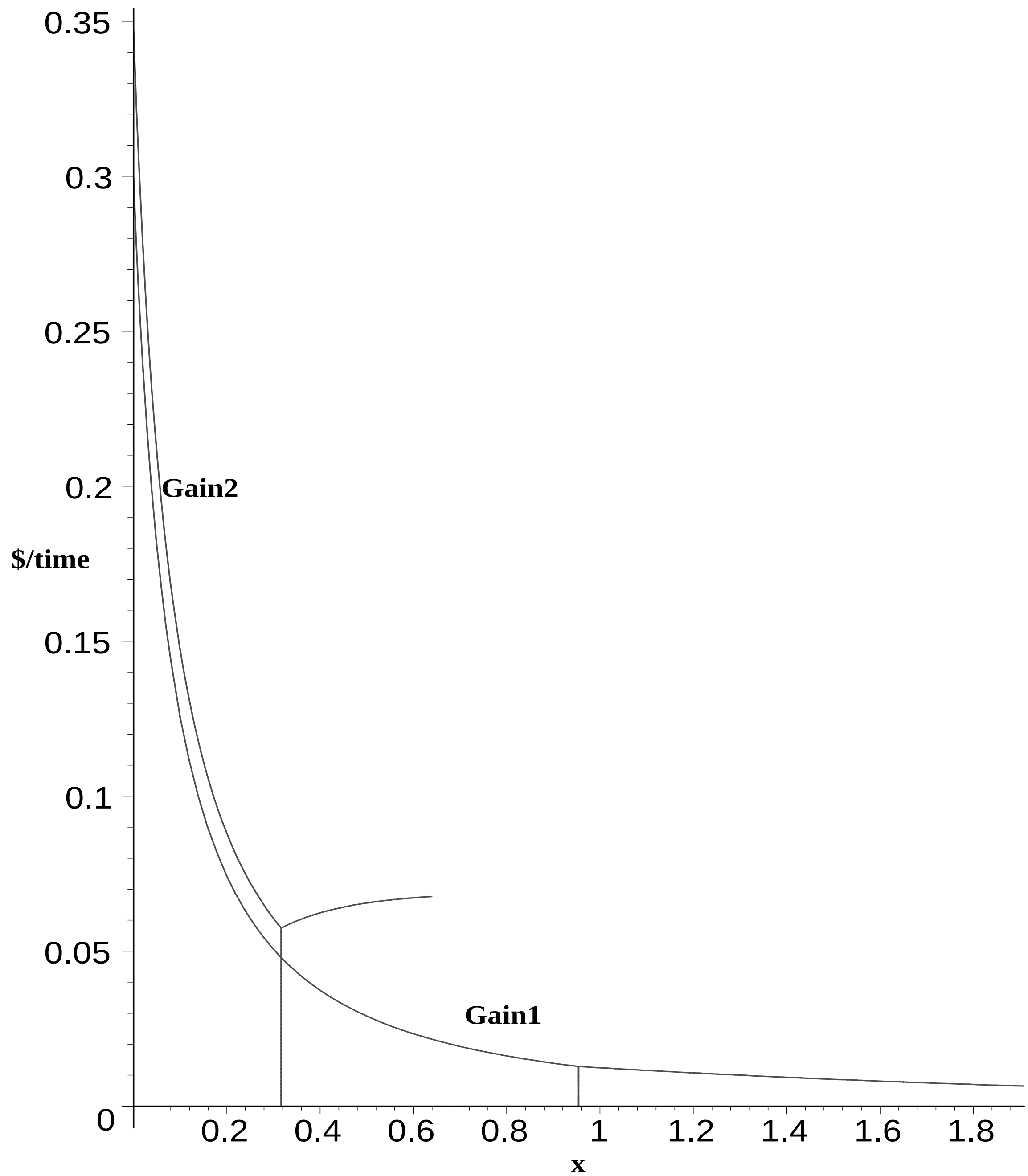

FIG. 5. Full profits less open access profits.

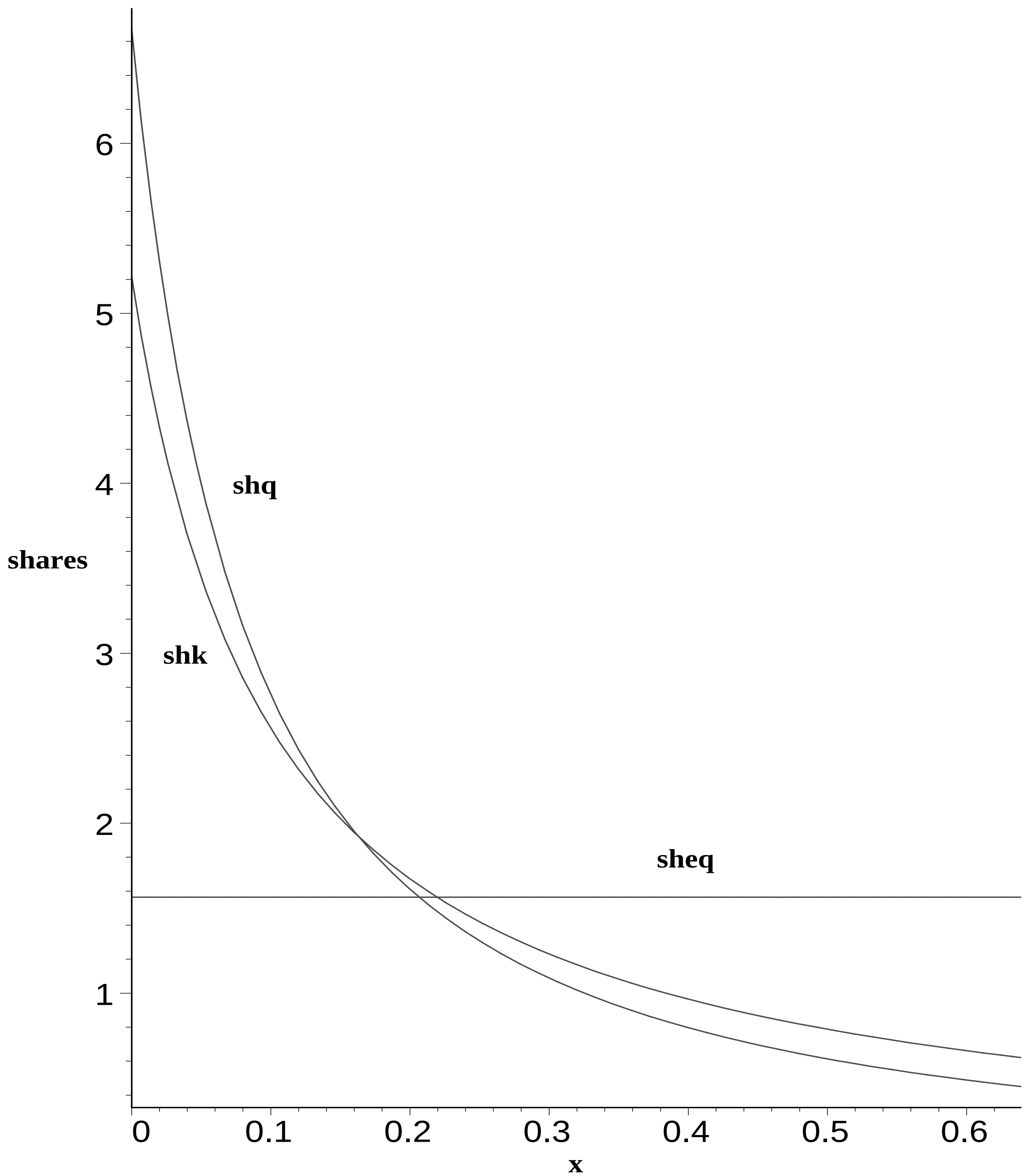

FIG. 6. Comparison of initial allocations.

Cobb Douglas Example

The Cobb Douglas example uses the production function

$$f = k^{\alpha}M^{\beta}/(1+mx)$$

The profit function is $(p-v)k^{\alpha}M^{\beta}/(1+mx) - rk - y$ and the optimal choice of k is given by

$$k^*(x)^{1-\alpha} = \frac{\alpha(p-v)M^{\beta}}{r(1+mx)}$$

The output of and profits earned by the x^{th} fisher with this choice of the input are

$$q(x) = (\frac{\alpha(p-v)}{r})^{\alpha/1-\alpha}(\frac{M^{\beta}}{!+mx})^{1/1-\alpha}$$

$$\pi(x) = (1-\alpha)(\frac{\alpha}{r})^{\alpha/1-\alpha}(\frac{(p-v)M^{\beta}}{!+mx})^{1/1-\alpha} - y$$

Putting $\pi(N) = 0$. this condition for open access equilibrium can then be written as

$$(p-v)M^{\beta}/(1+mN) = (\frac{y}{1-\alpha})^{1-\alpha}(\frac{r}{\alpha})^{\alpha} = \frac{y}{(1-\alpha)mz}$$

Using this condition $q(x) = mz(1+mN)^{\alpha/1-\alpha}M^{\beta}/(1+mx)^{1/1-\alpha}$ and the other equilibrium condition $\int_0^N q(x)dx = Q$ has the explicit solution

$$(1 + mN)^{\alpha/1-\alpha} - 1 = \frac{\alpha}{(1-\alpha)z} QM^{-\beta}$$

Note that as z increases. N and v decrease as the fishery becomes less profitable.

The full open access equilibrium will be denoted by t and for the purposes of illustration will be treated as a basic parameter together with m and z. From the first equilibrium condition ($v = 0$) and assuming that the natural growth function of the biomass is $g(M) = M(1 - M)$,

$$pt^{\beta} = \frac{y}{(1-\alpha)mz}(1 + mN_t)$$

where N_t is the number of vessels in the open access fishery.

For the fishery to be profitable at all, it must be the case that $t < 1$. As $t^{-\beta}(1 + mN_t)$ is decreasing in t, this requires that $p > y/((1 - \alpha)mz)$..

The first equilibrium condition can now be written as

$$p - v = \frac{pt^{\beta}(1 + mN)}{M^{\beta}(1 + mN_t)}$$

where N_t is the number of vessels in the open access fishery.

This formula can be used to show that the fishing profits of vessel x are

$$\pi(x) = (1-\alpha)mzpt^{\beta}\frac{((1+mN)/(1+mx))^{1/1-\alpha}-1}{(1+mN_t)}$$

The socially optimal biomass can now be calculated the condition given in Table I which becomes

$$\partial SS/\partial M = (p-v)(1-M) + v(1-2M) = 0$$

This can be rearranged as

$$\frac{(2M-1)M^{\beta}}{(2-\beta)M-(1-\beta)} = t^{\beta}\frac{1+mN}{1+mN_t}$$

The solution is increasing in z as the fishery becomes less profitable.

The rents earned in the fishery are

$$vM(1-M) = p(1-\frac{t^{\beta}(1+mN)}{M^{\beta}(1+mN_t)})M\,(1-M)$$

Using the formula $\partial N/\partial M = -(1+mN)^{1-2\alpha/1-\alpha}M^{-\beta}((2-\beta)M-(1-\beta))/(mz)$

$$p^{-1}\partial RR/\partial M = 1-2M+\frac{t^{\beta}(1+mN)}{M^{\beta}(1+mN_t)}((2-\beta)M-(1-\beta))(1+(1+mN)^{-\alpha/1-\alpha}M^{1-\beta}(1-M)/(mz))$$

so that rents are maximized by the biomass which solves

$$\frac{(2M-1)M^{\beta}}{(2-\beta)M-(1-\beta)} = \frac{t^{\beta}(1+mN)}{(1+mN_t)}(1+(1+mN)^{-\alpha/1-\alpha}M^{1-\beta}(1-M)/(mz))$$

which is greater than M^* provided $0 < t < 1$, that is that positive returns can be obtained from the fishery. The formula for $\partial RR/\partial M$ can be used to show that the $g\Gamma D$ in Table I is given by

$$g\Gamma D = \frac{t^{\beta}(1+mN)}{M^{\beta}(1+mN_t)}((2-\beta)M-(1-\beta))(1+mN)^{-\alpha/1-\alpha}M^{1-\beta}(1-M)/(mz)$$

Then the formula there for the derivative of full profits can be used to show that the sustainable full profit maximizing biomass satisfies

$$\frac{(2M-1)M^{\beta}}{(2-\beta)M-(1-\beta)} = \frac{t^{\beta}(1+mN)}{(1+mN_t)}(1+(1+mN)^{-\alpha/1-\alpha}M^{1-\beta}(1-M)(1-\frac{s(x)}{s_0(x)})/(mz))$$

From the formula for $q(x)$ above, the shares of the fishers in the total catch of the fishery are

$$s(x) = \frac{q(x)}{Q} = \frac{mz(1+mN)^{\alpha/1-\alpha}M^{\beta}}{(1+mx)^{1/1-\alpha}}$$

where $Q = M(1-M)$ in the long term.

The initial allocation of shares in the TAC based on historical catches is

$$s_{q0}(x) = \frac{mz(1+mN_t)^{\alpha/1-\alpha}t^{\beta}}{(1+mx)^{1/1-\alpha}}$$

.Finally, the shares based on historical input use are also proportional to $(1 + mx)^{1/1-\alpha}$ so must coincide with the historical catch shares.